Teaching and Learning Engineering Mathematics by Project Based Learning Method

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Rashmi Rani, Rajesh Kanna, Amer Abdelaal, Durai Pon Raj*
Al Ghurair University, UAE
*Corresponding author: prkanna@gmail.com**

Abstract

Teaching Mathematics in higher education is most challenging task for instructors. The objective of this paper is to improve teaching and learning engineering Mathematics via project-based learning. The major intention of this paper is to develop student's problem-solving skills using projects. This paper describes a method which will increase the interest of student to learn calculus. Students were asked to do a small project to emphasize the application of calculus. The paper emphasizes the importance of linking real time application of engineering calculations with Mathematics. In this paper, method to find surface area of curve rotating about an axis is taught through different theorem and further one curve is chosen to fabricate. This paper will demonstrate substantial difference between normal classroom teaching and project-based learning. This will enhance the teaching process of Engineering Mathematics among the students and faculty members. This paper will also allow the readers to develop new models to link various Mathematical calculations.

Keywords: Project based learning, Curve, Surface area, Mathematical theorem.

Introduction

This paper presents the conducted study on project-based learning (PBL) as an educational technique in teaching mathematical concepts. It has been recommended that teachers' instructional practices mediate the relationship between teacher background variables and student learning (Fennema and Franke, 1992). One instructional practice that has been supported to help students' Mathematical understanding is the use of instructional resources. Hiebert et al. (1997) points out three premises. The first, according to the author, is that when the contents are learned by understanding, they are flexible and can be adapted to the new situations and learning new concepts. The second is that learning mathematics stop being a practice focused on the memorization and application of algorithms, to become an investigative science, enabling the student to see how things work, how they relate to other topics and why they are this way. And finally, learning by understanding is an intellectually satisfying experience that provides confidence and involvement for students (Hiebert et al., 1997). Finding surface area is an important task in engineering practices. Packaging industries, surface painting, food industries, furniture manufacturing, textile industries and many more are required to find the surface area of an object. Introducing the complex Mathematics to engineering students to find surface area is one of the challenging tasks to the instructors. In the present paper, we have reported a novel method to find the surface area of a rotating curve through project-based learning. It is realized that this method excited students to learn Mathematics behind surface area calculation. Jaworski and Matthews (2011) have emphasized in their work about significance of learning mathematics at fundamental conceptual level. They exercised developmental research through student group activities. Mohamad Termizi Borhan (2011) presented problem-based learning. It is found that students and faculty have shown positive perception on problem-based learning. Clive L. Dym *et. al*. (2005) have reported that Project Based Learning (PBL) has more advantageous to learn Design course. They described in detail about design pedagogy and project-based learning. It should orient towards know-how and know-why models. This activity also results teamwork and life-long learning to the students. Katja Maass et al. (2011) discussed range of resources to improve instruction at large scale. Pardala et al. (2015) have focused on teaching mathematics and instructor challenges in three countries such as Poland, Kazakhstan and Russia. Sergei Abramovich et al. (2019) reported that the entire K-20 mathematics curriculum can be offered under a single umbrella through the techniques of concept motivation and action learning with real time applications. Also, they recommended to cooperate university faculty with school students. They emphasized on use of instructional resources and the teaching of mathematics.

BADA and Steve has explained, how Constructivism will give psychological effect on students' mind. Constructivism is an approach to teaching and learning based on the premise that learning is the result of "mental construction." In other words, students learn by fitting new information together with what they already know. Constructivists believe that learning is affected by the context in which an idea is taught as well as by students' beliefs and attitudes. Constructivism is a learning theory found in psychology which explains how people might acquire knowledge and learn. It therefore has direct application to education. The theory suggests that humans construct knowledge and meaning from their experiences (BADA and Steve, 2105).

The present work describes project-based learning method. The student was involved to fabricate a project relevant to the Mathematical topic. The model developed by the students help them to realize the application of Mathematics in real time situation. It is important in many engineering applications to know the surface area. There are mathematical methods and theorems to find area of surface of revolution of a solid generated by revolution about axes. The details are presented in the following section and further a fabricated project for the purpose of introducing PBL also discussed.

Method to measure Surface area of a solid generated by revolving a curve along axis

The first method to find surface area of a solid engendered by a continuous curve $y = f(x)$ by rotating the curve along axes. Theorem 1 explains the method to find the surface area generated by revolving an arc along any axes and example 1 describes the theorem extensively.

Theorem 1: Suppose $y = f(x)$ is a continuous function on [a, b] and has a continuous derivative on [a, b]. Then the area **S** of the surface of revolution by the arc of the curve $y = f(x)$ between x = a and x = b about the x-axis is

$$S = 2\pi \int_a^b |f(x)| \sqrt{1 + [f'(x)^2]}\, dx$$

$$= 2\pi \int_a^b |y| \sqrt{1 + [(\frac{dy}{dx})^2]}\, dx$$

Remark The corresponding area of the surface of revolution obtained by revolving an arc of a curve $x = g(y)$ from y = c to y = d about the y-axis is

$$S = 2\pi \int_c^d |g(y)| \sqrt{1 + [g'(y)^2]}\, dy$$

$$= 2\pi \int_c^d |x| \sqrt{1 + [(\frac{dx}{dy})^2]}\, dy$$

Example 1: Find the surface area of the solid generated by revolving the curve $x^2 = 8y$ from (0, 0) to (4, 2).

The graph of given parabolic curve is shown in Figure 1.

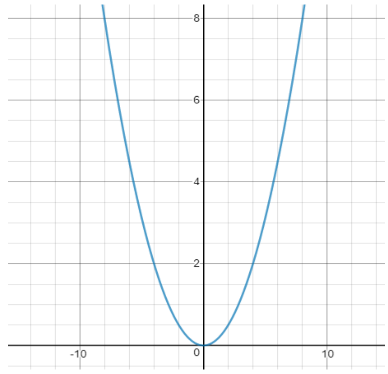


Figure 1. Surface of the solid generated by parabolic curve

Differentiating both sides of the equation $x^2 = 8y$ with respect to y, we have

$$2x\frac{dx}{dy} = 8 \Longrightarrow \frac{dx}{dy} = \frac{4}{x} = \frac{4}{2\sqrt{2y}} = \sqrt{\frac{2}{y}}$$

Now the surface of the solid generated by arc from y = 0 to y = 2 is given as

$$S = 2\pi \int_a^b |x| \sqrt{1 + [(\frac{dx}{dy})^2]}\, dy$$

$$= 2\pi \int_0^2 2\sqrt{2y} \sqrt{1 + [(\sqrt{\frac{2}{y}})^2]}\, dy$$

$$= 4\pi\sqrt{2} \int_0^2 \sqrt{y} \sqrt{\frac{y+2}{y}}\, dy$$

$$= 4\pi\sqrt{2} \int_0^2 \sqrt{y+2}\, dy$$

$$= 4\pi\sqrt{2} \left[\frac{(2+y)^{3/2}}{3/2}\right]_{y=0}^{y=2}$$

$$= \frac{32}{3}\pi\left(2^{3/2} - 1\right)$$

= 61.29584266 square unit.

The next theorem will describe the method to find the surface area of a portion of curve with parametric equation.

Theorem 2: If a portion of the curve of parametric equations x = x(t), y = y(t) between the points corresponding to t_1 and t_2 is revolved about the x-axis, the surface area **S** is

$$S = 2\pi \int_{t_1}^{t_2} |y(t)| \sqrt{[x'(t)^2] + [y'(t)^2]}\, dt.$$

Example based on theorem 2 is given in example 2 where a parametric curve has selected to find the surface area generated by the curve along any axis.

Example 2: Find the surface area of the solid generated by the revolution of asteroid $x^{2/3} + y^{2/3} = 8^{2/3}$, about x-axis.

Solution: The Graph of given curve is shown in Figure 2.

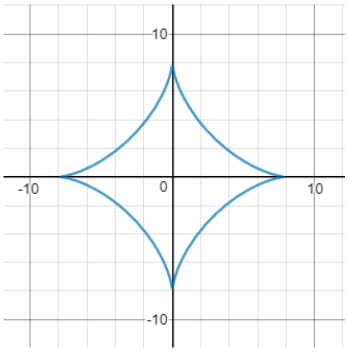


Figure 2. Solid generated by the revolution of asteroid

The parametric equation of the given asteroid is
$x = 8\cos^3 t$, and
$y = 8 sin^3 t$, where t is from 0 to 2π

$\Longrightarrow \frac{dx}{dt} = -24\cos^2 t \, \sin t$ and $\frac{dy}{dt} = 24\sin^2 t \, \cos t$

Therefore,

$$\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} = \sqrt{576 \, \cos^4 t \, \sin^2 t + 576 \, \cos^4 t \, \sin^2 t}$$

$$= 24\sin t \cos t \sqrt{\cos^2 t + sin^2 t}$$

$$= 24\sin t \, \cos t$$

So, the generated surface area is given as

$$S = 2\int_0^{\pi/2} 2\pi y \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$$

$= 4\pi \int_0^{\pi/2} 8 \, sin^3 t \, 24\sin t \, \cos t \, dt$

$= 768\pi \int_0^{\pi/2} sin^4 t \cos t \, dt$

$= 768\pi \, (\frac{sin^5 t}{5})_{t=0}^{t=\pi/2}$ [Applying integration by Substitution]

$= 768\pi$ square unit.

Theorem 3 provides an idea to find the surface area of a solid generated by a polar curve by revolution and example 3 is an explanation of theorem 3.

Theorem 3 Suppose $r = f(\theta)$ is a polar curve, then the area of the surface of revolution generated between the angle $\theta = \alpha$ to $\theta = \beta$ about initial line is given as

$$S = 2\pi \int_\alpha^\beta |y| \sqrt{r^2 + (\frac{dr}{d\theta})^2} \, d\theta,$$

where $y = r\sin\theta$ and $x = r \, \cos\theta$

Example 3 Find the area of revolution generated by revolving $r = 8\cos\theta$ about the initial line.

Solution: The graph of curve is given in Figure 3.

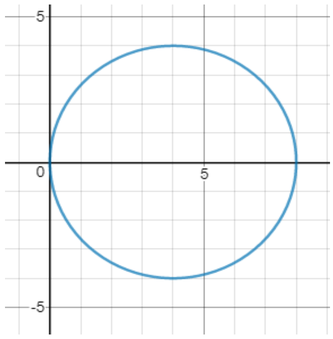


Figure 3. Surface area of a solid generated by a polar curve

The given curve is $r = 8\cos\theta$

$\Longrightarrow \frac{dr}{d\theta} = -8\sin\theta$ and

$$\sqrt{r^2 + (\frac{dr}{d\theta})^2} = \sqrt{64\,(\cos^2 t + sin^2 t)} = 8$$

So, the surface area of the solid generated by the arc from $\theta = 0$ to $\theta = \pi/2$ is

$S = 2\pi \int_0^{\pi/2} |y| \sqrt{r^2 + (\frac{dr}{d\theta})^2} \, d\theta$, where $y = r\sin\theta$

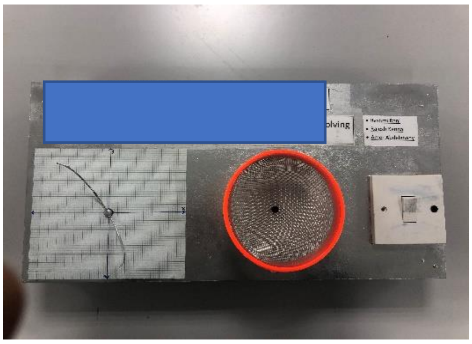
$= 128\pi \int_0^{\pi/2} \sin\theta \, \cos\theta \, d\theta$

$=64\pi$ square unit.

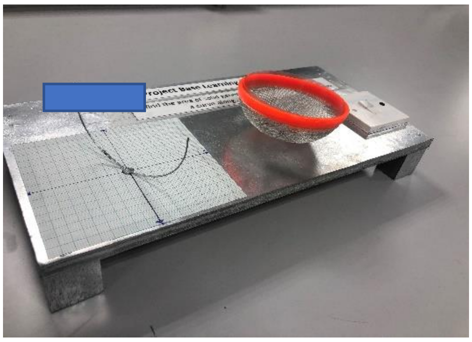
Fabricated model to explore Surface area

It is investigated the influence of use of instructional resources on the teaching of mathematics as a mediator of the relationship between teacher variables and students' mathematics learning (Besong Francis, 2108). Although learners' belief about innovations is important, without the teacher's positive support, learners may find it difficult to explore instructional resources in mathematics instruction desired to enhance their learning outcomes. It is reasonable to suggest that the use of instructional resources in mathematics instruction is the best teaching strategy to facilitate learners' active engagement in the lesson. Similarly, the use of instructional resources in mathematics instruction has been claimed to ease the teacher's workload and enhance teacher and learners' effectiveness in the classroom (Besong Francis, 2108).

In this paper, student is encouraged to fabricate a model to explore the mathematics behind it. The designed project was used to demonstrate in the classroom. Hence it is decided to be portable size of 75 cm x 50 cm x 10 cm. A metal wire of 20 cm was bent to form a curve and fixed on a bearing mounted on the board. A shaft connects the metal wire and an electric motor which was powered by a battery to rotate when the switch is on. A graph sheet is pasted at the surface to give visibility to the student when they view this wire is rotating. The details are shown in the Figure 4 in which a continuous curve is revolved along y-axis. A battery powered motor is provided to revolve the wire frame which is mounted on the graph sheet. When it revolves about y axis the resultant surface will be appear as shown in Fig. 4.b and demonstrated to the students. Then the surface area of generated solid was found using Theorem 1. In similar ways, students can fabricate any model in future and can be able to find surface area using Theorem 1, Theorem 2 or Theorem 3. This method allows the student to view the rotation of wire frame results surface area and enhance the involvement of student. Also, this model will provide substantial understandings on the influence of the use of instructional resources in the teaching of mathematics on students' performance in mathematics. This study clearly indicates the excitement of student for learning mathematics in connection with this project which reflects real time applications of mathematics.



a) Top view



b) side view

Figure 4. Developed teaching aid for demonstrating surface area calculation

Results and Discussion

The project-based learning was introduced during fall semester to the college of engineering students at our university. One of the author, Mr. Amer who is mechanical engineering student involved in fabricating this project model. Once the model was fabricated, it was presented to a group of 5 students from different specialization. All the students were excited and showed interest to practice the model and showed interest to learn the application mathematics behind this model. They realize that by using mathematics they could find surface area of any curve. Hence the linkage between engineering application and mathematics was demonstrated through the project-based learning. Further many student groups had shown interest on fabricating other special geometry surface area and to learn relevant mathematics. For instance, to measure surface area of package material useful in electronic industry is another topic of interest for another group of students in near future. The objective of this work was accomplished by developing new teaching techniques to meet the needs of students in classroom and enhance their knowledge and problem-solving skills via projects.

Conclusion

Project based learning is an educational technique that endorses students' knowledge of intellectual mathematical concept. The method engages students' interest and curiosity by allowing them to fabricate the project that will relate to real-world situation (Chin William, 2014). The aim of this paper is to motivate students towards more conceptual understanding of mathematics through their participation in a project-based learning. We have attempted to make links between the innovatory practices of the project and the theory proposed explicitly. The present paper presented a project model useful for demonstrating calculation of surface area of by mathematical method and its real time applications. It allows the readers to explore to develop new model to demonstrate surface area calculation by fabricating projects with the involvement of students.

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