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The Marginal Stability of Oscillatory Bénard-Marangoni Convection With Internal Heat Generation

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HIGHLIGHTS

- Any direction of temperature difference-induced motion of a geometric fluid
- The internal heat source is a factor in the fluid layer's temperature increase.
- The oscillating motion phenomena are influenced by gravity

ABSTRACT

The onset of oscillatory Benard-Marangoni convection in a horizontal fluid layer with internal heat generation and a deformable free surface is studied using an analytical technique employing the classical linear stability theory. We consider the case when both the Rayleigh number and Marangoni number are linearly dependent. We obtained the analytical result for the expansion of the Rayleigh number in the limit of a very short wave. We found that the internal heat generation factor influences the leading order of Rayleigh number.

Keywords: *Bénard-Marangoni convection, Rayleigh-Bénard convection, Marangoni convection, free convection*

INTRODUCTION

Convection is a type of energy diffusion process that occurs in a fluid and is important to a physical state. Convection is simply the movement of a geometric fluid caused by a temperature difference in any direction. If we examine the observations, we will discover that convection phenomena occur all around us, either naturally (free convection), as in geophysics (Knopoff, 1969; Plummer & McGeary 1991), or caused by human actions (forced convection), as in the formation of crystals in semiconductor production (Elliot, 1998; Schwabe, 1988; Ostrach, 1983).

Wilson (1993) was the first to investigate convection involving internal heat generation, obtaining an analytical representation of the Marangoni number, M, and was followed by Char and Chiang (1994), who investigated the effect of rotation and internal heat generation at the beginning of Benard-Marangoni oscillating convection. The short-wave asymptotic analysis investigation of the onset of Marangoni oscillating convection with internal heat generation performed by Hashim (2001) using a single-layer model served as the inspiration for the work we carried out.

METHODOLOGY

The mathematical modeling of the fluid is developed based on a single-layer fluid model as shown in Figure 1.

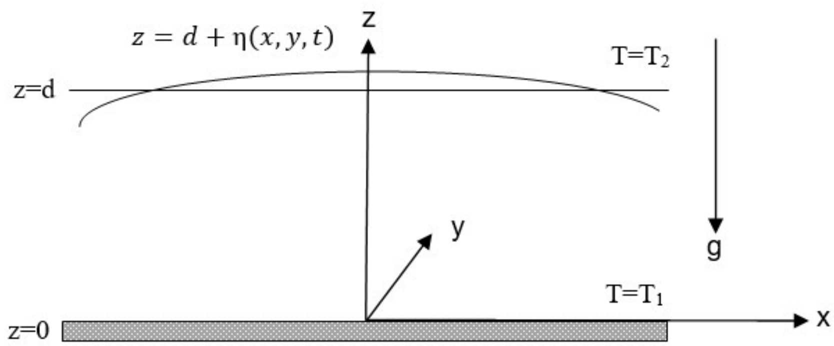

Figure 1: Single Layer Fluid Model

A dimensionless linear equation for the case of the lower boundary layer with perfect heat conduction and affected by the internal heat generation factor based on the fluid model in Figure 1 can be expressed as follows:

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{s}{P_r}\right)W = a^2 R\Theta, \tag{1}$$

$$(D^2 - a^2 - s)\Theta + [1 - Q(1 - 2z)]w = 0, \tag{2}$$

with the boundaries conditions $W = \vec{0}$, $DW = \vec{0}$ dan $\theta = \vec{0}$ pada $z = 0$ while the boundary conditions on the surface are free $z = d$ can be expressed as follows:

$$sf - W = 0, \tag{3}$$

$$C_r\left(D^2 - 3a^2 - \frac{s}{P_r}\right)DW - a^2(a^2 + B_0)f = 0, \tag{4}$$

$$(D^2 + a^2)W + a^2\,\Gamma R(1 + Q)f] = 0, \tag{5}$$

$$D\Theta + B_i[\Theta - (1 + Q)f] = 0. \tag{6}$$

with $\Gamma R = M$. Operator $D = d/dz$ represents the differentiation of the vertical component z. Quantity $W = W(z), \Theta = \Theta(z)$ dan f is the amplitude of the velocities in the vertical direction, the temperature in the vertical direction, and the deformable on the free surface. Parameter $a = \left(a_x^2 + a_y^2\right)^{0.5}$ represents the overall horizontal wave number and the parameter s is referred to as the growth rate with $Ny(s)$ represents the growth rate of instability and $Kh(s) = \omega$ represents the frequency. The dimensionless numbers used in this research are Biot number $B_i = hd/k$, Bond number $B_0 = \rho_0 g d^2/\tau_0$, Crispation number $C_r = \rho_0 \nu\kappa/\tau_0 d$, internal heat number $Q = qd^2/2\kappa\Delta T$, Marangoni number $M = \gamma d\Delta T/\rho_0\nu$, Prandtl number $P_r = \nu/\kappa$ and Rayleigh number $R = q\alpha d^3\Delta T/\nu\kappa$.

Linearized Dimensionless Equations

The solution for $W(z), \Theta(z), f$ and R are obtained from a linear equation without parameters by considering the imaginary quantity of the fluid velocity growth rate $kh(s) = \omega$ in the limit $a \to \infty$ for the case $C_r \neq 0$. The system is said to be stable when the manipulation is shrinking over time if the real element of growth rate $Ny(s) < 0$ and vice versa when $Ny(s) > 0$. The onset of instability occurs when $Ny(s) = 0$ and the oscillation that occurs is steady if $Kh(s) = 0$ and oscillates if $Kh(s) \neq 0$ as in our study.

Numerical solutions for R and ω in the limit $a \to \infty$ for the case of a turbulent fluid surface ($C_r \neq 0$) have suggested the formation of a thin fluid layer of thickness $O(1/a)$ near the boundary $z = 1$ whose coordinates can be written as $1 - z = Z/a$ (Hashim & Wilson, 1999). Motivated by this statement, we assume that the asymptotic solutions of $W(z), R,$ and ω are of the following forms:

$$W(Z) = \sum_{i=0}^{\infty} \varepsilon^i W_i(Z), \tag{7}$$

$$R = \varepsilon^{-4} \sum_{i=0}^{\infty} \varepsilon^i R_i, \tag{8}$$

$$\omega = \varepsilon^{-3} \sum_{i=0}^{\infty} \varepsilon^i \omega_i, \tag{9}$$

while the asymptotic solutions for Θ and f are stated as follows:

$$\Theta(Z) = \sum_{i=0}^{\infty} \varepsilon^i \Theta_i(Z), \tag{10}$$

$$f = \varepsilon^2 \sum_{i=0}^{\infty} \varepsilon^i f_i. \tag{11}$$

Solving the governing equations (1) dan (2) with $Kh(s) = \omega$ yields

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{i\omega}{P_r}\right)(D^2 - a^2 - i\omega)W + a^2 R[1 - Q(1 - 2z)]W = 0. \tag{12}$$

In terms of Z, equation (11) can be written as follows:

$$\left(\widehat{D}^2 - 1\right)^3 W + a^{-4} R\left[1 - Q\left(\frac{2Z}{a} - 1\right)\right] W = 0. \tag{13}$$

Θ and f are stated as follows:

$$\Theta = \frac{a^2}{R}\left(\widehat{D}^2 - 1\right)\left(\widehat{D}^2 - 1 - \frac{i\omega}{a^2 P_r}\right) W, \tag{14}$$

$$f = -\frac{aC_r}{(B_0 + a^2)}\left(\widehat{D}^2 - 3 - \frac{i\omega}{P_r}\right)\widehat{D}W, \tag{15}$$

with $\varepsilon = a^{-0.5}$ dan $\widehat{D} = d/dz = -D/a$ (Hashim & Wilson, 1999). The boundary conditions at $Z = 0$ are as follows:

$$i\omega f - W = 0, \tag{16}$$

$$(\widehat{D}^2+1)W+\Gamma R[\Theta-(Q)f]=0, \tag{17}$$

$$-a\widehat{D}\Theta+B_i[\Theta-(1+Q)f]=0, \tag{18}$$

While for the condition $Z\to\infty$ obtained $W\to 0$, $\widehat{D}W\to 0$ dan $\Theta\to 0$.

FINDINGS AND DISCUSSIONS

Solving equation (12) and the boundaries conditions (13)−(18) by using MAPLE yields the solution for leading order for $z\to\infty$ as

$$W_0(Z)=AZe^{-Z} \tag{19}$$

where A is an arbitrary constant. Figure 2 shows the fluid speed limit suit curve $W_0(Z)$ in the limit $a\to 10$ and $a\to 100$ which has been plotted from equation (19). It is found that the velocity of the fluid increases when crossing the depth d and decreases sharply when very close to the surface of the fluid. This phenomenon is only a transient effect of fluid velocity seen in the main stage. Solving equations (14) and (15) gives $f_0=0$ and $\Theta_0=0$.

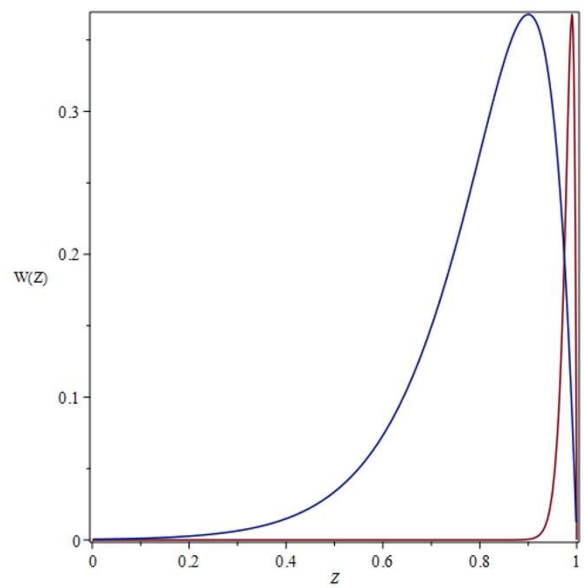

Figure 2: $W_0(z)$ respect to z when $a\to 10$ dan $a\to 100$

CONCLUSION AND RECOMMENDATIONS

As a result of the analysis, we discovered that the internal heat source contributes to the rise in temperature of the fluid layer, causing the system to become unstable. We also discovered that gravity influences the phenomenon of oscillating motion.

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CONFLICT OF INTEREST DISCLOSURE

The authors declared that they have no conflicts of interest to disclose.

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