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STATISTICAL INFERENCE ON SINE-EXPONENTIAL DISTRIBUTION PARAMETER

[1]Akeem Ajibola Adepoju, [2]Akanji Olalekan Bello, [3]Alhaji Modu Isa, [4]Akinrefon Adesupo & [5]Jamiu S. Olumoh

[1]Department of Statistics,
Faculty of Computing and Mathematical Sciences, Aliko Dangote University of Science and Technology, Nigeria
[2]Department of Statistics,
Ahmadu Bello University, Zaria, Nigeria
[3]Department of Mathematics and Computer Science,
Borno State University, Maiduguri, Nigeria
[4]Department of Statistics, Faculty of Physical Sciences Modibbo Adama University, Adamawa, Nigeria
[5]Department of Mathematics and Statistics,
American University of Nigeria, Nigeria

[1]Corresponding author: akeebola@gmail.com

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ABSTRACT

The Sine-Exponential (Sine-E) distribution is a probability distribution that combines the periodic behavior of the sine function with the decay characteristic of the exponential function. This study addresses the problem of identifying the most accurate and reliable estimation method for the parameter of the Sine-E distribution. The objective is to evaluate various parameter estimation techniques, including Maximum Likelihood Estimation (MLE), Least Squares Estimation (LSE), Weighted Least Squares Estimation (WLSE), Maximum Product of Spacing Estimation (MPSE), Cramer-von-Mises Estimation (CVME), and Anderson-Darling Estimation (ADE), using Mean Square Error (MSE) as the criterion for determining the technique with the minimum error. The study's findings reveal that as sample size increases, the parameter estimates for all techniques converge to the true parameter value, with decreases in bias, MSE, and mean relative estimates. Among the techniques evaluated, the MPSE method consistently provides estimates closest to the true parameter value and exhibits the least bias and lowest MSE across small, moderate, and large sample sizes, making it the best estimator for the Sine-E distribution.

Keywords: Sine-Exponential Distribution, Maximum Product, Cramer-von-Mises, Anderson-Darling, Mean Square Error.

INTRODUCTION

Sine-Exponential (Sine-E) is of great importance in statistical modeling despite its shortcomings in terms of constant failure rate and memoryless condition. It stands among the most utilized models. Areas of its applicability include but are not limited to reliability theory, queuing theory, population growth, radioactive decay, and the amount of medicine in the bloodstream. Sine distribution is probability distribution based on a portion of the sine curve. The sine distribution is capable of fitting multimodal datasets. In order to achieve a better model of extending the capacity of distribution with the inability to model multimodal, the sine family was introduced by Kumar et al. (2015) with its probability density function (pdf), cumulative distribution function (cdf), and other mathematical properties. Thereafter, Isa et al. (2022) explored the sine family of distribution to extend the exponential distribution. Maximum likelihood was adopted to estimate the parameter, and the breaking stress of the carbon fiber dataset was adopted to justify the model's flexibility. However, no inference about the model parameter was presented. Authors who extend the exponential distribution include Adepoju et al. (2023), Gul et al. (2023), Balogun et al. (2021), Bello et al. (2020, 2021), Ibrahim et al. (2020a), Ibrahim et al. (2020b), Kajuru et al. (2023), and other extended distribution include Isa et al. (2023) and Adepoju et al. (2024).

Researchers employed inferential techniques to study the various methods that provide estimates of model parameter(s). These techniques are employed in simulation studies and are also adopted in real-life applications. The Çetinkaya (2022) studied the generalized fiducial inference for the shape parameters of the Chen distribution. Simulation and application using datasets were conducted to prove that the generalized fiducial inference performs better than the other estimators for the shape parameters of the Chen distribution. Meanwhile, Adepoju et al. (2024) compared six techniques for estimating Type I Half Logistic Topp-leone Exponential distribution through simulation, and the Maximum Product of Spacing Estimation (MPSE) technique proved to be better for estimating the model. Migdadi et al. (2023) focused on inference for power Rayleigh distribution parameters via Bootstrap, Likelihood, and Bayesian estimation methods. Notably, simulation studies and real-life applications of these methods were considered to examine the performance of the estimates by various estimators. The results revealed that both interval and point estimates are efficient and capable of estimating the model parameters, with Bayesian as the most preferred method for point and interval estimates under the General Entropy GE loss function. At the same time, Anabike et al. (2023) introduced the Zubair-Exponential model. Simulation studies using various classical approaches and Bayesian methods were conducted. Accordingly, the model was applied to the survival times of Guinea pigs using various estimation methods, and the model's efficacy was demonstrated.

Other articles that explored various estimation techniques for distinct models include Hassan et al. (2023), Alotaibi et al. (2023), Yilmaz et al. (2021), Adepoju, Usman et al. (2021), Adepoju, Isa et al. (2021), ZeinEldin et al. (2019), Warsono et al. (2019), Cheng and Amin (1979), Anderson and Darling (1952), Macdonald (1971), and Swain et al., (1988), to name a few.

This article aims to investigate the behavior of various estimators of Sine-E model parameters using different classical techniques. The techniques considered in this article are the Maximum Likelihood Estimation (MLE), Least Squares Estimation (LSE), Weighted Least Square Estimation (WLSE), MPSE, Cramér–von Mises Estimation

(CVME), and Anderson-Darling Estimation (ADE). The motivation for this study lies in the consistency and efficiency assessment of the model parameter's estimators for Sine-E distribution at different parameter values across the sample sizes.

ESTIMATION TECHNIQUES

This section introduced the cdf and pdf of the Sine E Distribution and various techniques of estimation of the model parameter

$$F_{Sin-E}\left(x;\alpha\right)=\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x}\right)\right), \tag{1}$$

$$f_{Sin-E}\left(x;\alpha\right)=\frac{\pi}{2}\alpha e^{-\alpha x}\cos\left(\frac{\pi}{2}\left(1-e^{-\alpha x}\right)\right). \tag{2}$$

Maximum Likelihood Estimation (MLE)

MLE is a popular technique for parameter estimation. The technique will be considered among other techniques for estimating the parameters of the Sine-E model. Now, consider a random sample X_i where $i=1,...,n$, obtained from the Sine-E distribution parameterized with α. The log-likelihood function $L\left(\Psi\right)$ of equation (2) can be obtained as

$$L\left(\Psi\right)=m\log\left(\frac{\pi}{2}\right)+m\log\alpha-\alpha\sum_{i=0}^{n}x_{(i)}+\sum_{i=0}^{n}\log\cos\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right). \tag{3}$$

Differentiating $L\left(\Psi\right)$ in equation (3) with respect to and setting the result to zero will provide the estimators. Thus,

$$\frac{\delta L\left(\Psi\right)}{\delta\alpha}=\frac{m}{\alpha}+\sum_{i=0}^{m}x_{(i)}-\sum_{i=0}^{m}\tan\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right)=0. \tag{4}$$

Least Square Estimation (LSE)

The LSE technique is explored in this study. The LSE of the Sine-E distribution with parameter α can be obtained by minimizing the equation (5) with respect to the parameter. The LSE function can be

$$LSE_{\Psi} = \sum_{i=1}^{m}\left[F_{\text{Sin}-E}\left(x_{(i)};\alpha\right) - \frac{i}{m+1}\right]^2 = \sum_{i=1}^{m}\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right) - \frac{i}{m+1}\right]^2 \quad (5)$$

Thus, the LSE can be obtained by differentiating the equation [5] with respect to the parameter α and setting the result to zero

$$\frac{\delta LSE_{\Psi}}{\delta\alpha} = 2\sum_{i=1}^{m}\zeta_i^{(\alpha)}\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right) - \frac{i}{m+1}\right] = 0 \quad (6)$$

where $$\zeta_i^{(\alpha)} = \frac{\pi x e^{\alpha x}\cos\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right)}{2}, \quad (7)$$

More details are provided by Swain et al (1988).

Weighted Least Square Estimation (WLSE)

Another technique is the WLSE. Now, the WLSE of the Sine-E distribution parameter α is obtained by minimizing the equation below with respect to parameter α The WLSE function can be defined as

$$WLSE_{\Psi} = \sum_{i=1}^{m}\left(\frac{(m)^2(m+1)}{i(m+1-i)}\right)\left[F_{\text{Sin}-E}\left(x_{(i)};\alpha\right) - \frac{i}{m+1}\right]^2, \quad (8)$$

$$= \sum_{i=1}^{m}\left(\frac{(m+1)^2(m+1)}{i(m+1-i)}\right)\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right) - \frac{i}{m+1}\right]^2,$$

$$\frac{\delta WLSE_{\Psi}}{\delta\alpha} = 2\sum_{i=1}^{m}\zeta_i^{(\alpha)}\left(\frac{(m+1)^2(m+1)}{i(m+1-i)}\right)\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right) - \frac{i}{m+1}\right], \quad (9)$$

where $\zeta_i^{(\alpha)}$ is defined in equation (7). More details are provided by Swain et al (1988).

Maximum Product of Spacings Estimates (MPSE)

The MPSE technique is another vital technique introduced by Cheng, et al. (1979). The MPSE estimate of the Sine-E distribution parameter can be obtained by maximizing the equation below with respect to the parameter:

$$MPS_{\Psi} = \frac{1}{m+1}\sum_{i=1}^{m+1}\log\left(Z_i\right), \quad (10)$$

where

$$Z_i = F_{\text{Sin}-E}\left(x_{(i)};\alpha\right) - F_{\text{Sin}-E}\left(x_{(i-1)};\alpha\right),$$

$$F_{\text{Sin}-E}\left(x_{(0)};\alpha\right) = 0, \quad F_{\text{Sin}-E}\left(x_{(m+1)};\alpha\right) = 1,$$

and

$$\sum_{i=1}^{m+1} Z_i = 1.$$

Thus, the MPS_{Ψ} estimate is obtained by differentiating the equation (10) with respect to the parametee where $F_{\text{Sin}-E}\left(x_{(i)};\alpha\right)$ is the cdf of the Sine-E distribution defined in (1)

Cramér-von-Mises Estimates (CVME)

The CVME technique is another vital estimation technique that was considered in this study. The concept of this technique is to minimize the function in equation (11) with respect to parameter α .The CVME distance function for Sine-E distribution can be expressed as

$$\begin{aligned} CVM_{\Psi} &= \frac{1}{12m} + \sum_{i=1}^{m}\left[F_{\text{Sin}-E}\left(x_{(i)};\alpha\right) - \frac{2i-1}{2m}\right]^2 \\ &= \frac{1}{12m} + \sum_{i=1}^{m}\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right) - \frac{2i-1}{2m}\right]^2. \end{aligned} \quad (11)$$

Thus, the estimate of the Sine-E distribution parameter using the CVM technique can be obtained by differentiating the equation (11) with respect to α .and setting the result to zero.

$$\frac{\delta CVM_{\Psi}}{\delta\alpha} = 2\sum_{i=1}^{m}\zeta_i^{(\alpha)}\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x}\right)\right) - \frac{2i-1}{2m}\right]^2 = 0, \quad (12)$$

where $\zeta_i^{(\alpha)}$ is defined in (7). More details are provided by Macdonald (1971).

Anderson–Darling Estimation (ADE)

The ADE technique is also considered in this study. Applying the ADE technique for the Sine-E distribution parameter α .

$$ADE_{\Psi} = -m - \frac{1}{m}\sum_{i=1}^{m}(2i-1)\left\{\log\left[F_{\text{Sin}-E}\left(x_{(i)};\alpha\right)\right] + \log\left[1 - F_{\text{Sin}-E}\left(x_{(m+1-i)};\alpha\right)\right]\right\},$$
$$= -m - \frac{1}{m}\sum_{i=1}^{m}(2i-1)\left[\log\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right)\right] + \log\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(n+1-i)}}\right)\right)\right]\right], \quad (13)$$

Thus, the estimates can be easily obtained by differentiating equation (13) with respect to α . and setting the results to zero.

$$\frac{\delta ADE_{\Psi}}{\delta\alpha} = -\frac{1}{m}\sum_{i=1}^{m}(2i-1)\left[\frac{\zeta_i^{(\alpha)}}{\left[\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(i)}}\right)\right)\right]} - \frac{\zeta_{n+1-i}^{(\alpha)}}{\left[1-\sin\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(n+1-i)}}\right)\right)\right]}\right] = 0 \quad (14)$$
,

$$\zeta_{n+1-i}^{(\alpha)} = \frac{\pi x_{(n+1-i)} e^{\alpha x_{(n+1-i)}} \cos\left(\frac{\pi}{2}\left(1-e^{-\alpha x_{(n+1-i)}}\right)\right)}{2}$$

where $\zeta_i^{(\alpha)}$ is defined in equation (7) and

Details are provided by Anderson and Darling. (1952)

SIMULATION STUDY

Now, the performance of the MLE, LSE, WLSE, MPSE, CVME, and ADE are investigated for the Sine-E distribution parameter through a simulation study while considering 10,000 replications. Data were generated with different sample sizes (10, 30, 50, 75, 100). The estimates, bias, Mean Square Error (MSE), and mean relative estimate were obtained by R software. Thus, it is obtained as follows.

Table 1

Estimates of Various Estimation Techniques for Parameter Lambda = 0.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.5559	0.5383	0.5349	0.4966	0.5469	0.5337
30	0.5206	0.5180	0.5165	0.4932	0.5210	0.5159
50	0.5077	0.5057	0.5051	0.4893	0.5075	0.5046
75	0.5030	0.5010	0.5008	0.4896	0.5022	0.5006
100	0.5043	0.5021	0.5023	0.4936	0.5031	0.5021

Table 2

Bias of Various Estimation Techniques for Parameter Lambda = 0.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.0559	0.0383	0.0349	-0.0034	0.0469	0.0337
30	0.0206	0.0180	0.0165	-0.0068	0.0210	0.0159
50	0.0077	0.0057	0.0051	-0.0107	0.0075	0.0046
75	0.0030	0.0010	0.0008	-0.0104	0.0022	0.0006
100	0.0043	0.0021	0.0023	-0.0064	0.0031	0.0021

Table 3

Mean Square Error of Various Estimation Techniques for Parameter Lambda = 0.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.0354	0.0438	0.0406	0.0259	0.0452	0.0345
30	0.0090	0.0121	0.0110	0.0077	0.0123	0.0102
50	0.0045	0.0061	0.0054	0.0042	0.0061	0.0052
75	0.0029	0.0038	0.0035	0.0029	0.0039	0.0034
100	0.0023	0.0030	0.0027	0.0022	0.0030	0.0026

Table 4

Mean Relative Estimates of Various Estimation Techniques Lambda = 0.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.2660	0.2908	0.2803	0.2418	0.2942	0.2664
30	0.1473	0.1671	0.1590	0.1411	0.1680	0.1552
50	0.1044	0.1214	0.1145	0.1036	0.1216	0.1130
75	0.0844	0.0972	0.0922	0.0852	0.0973	0.0913
100	0.0758	0.0863	0.0818	0.0755	0.0864	0.0813

Table 5

Estimates of Various Estimation Techniques for Parameter Lambda = 1.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	1.6676	1.6150	1.6047	1.4897	1.6407	1.6011
30	1.5618	1.5539	1.5495	1.4796	1.5630	1.5477
50	1.5231	1.5170	1.5153	1.4680	1.5224	1.5138
75	1.5090	1.5029	1.5025	1.4687	1.5066	1.5018
100	1.5129	1.5064	1.5068	1.4809	1.5092	1.5062

Table 6

Bias of Various Estimation Techniques for Parameter Lambda = 1.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.1677	0.1150	0.1047	-0.0103	0.1407	0.1011
30	0.0619	0.0539	0.0495	-0.0204	0.0630	0.0477
50	0.0231	0.0170	0.0153	-0.0320	0.0224	0.0138
75	0.0090	0.0029	0.0025	-0.0313	0.0066	0.0018
100	0.0129	0.0064	0.0068	-0.0191	0.0092	0.0062

Table 7

Mean Square Error of Various Estimation Techniques for Parameter Lambda = 1.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.3190	0.3942	0.3652	0.2332	0.4066	0.3108
30	0.0809	0.1088	0.0991	0.0696	0.1107	0.0921
50	0.0402	0.0546	0.0487	0.0380	0.0551	0.0470
75	0.0264	0.0346	0.0313	0.0261	0.0348	0.0306
100	0.0207	0.0268	0.0243	0.0201	0.0269	0.0238

Table 8

Mean Relative Estimates of Various Estimation Techniques Lambda = 1.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.2660	0.2908	0.2803	0.2418	0.2942	0.2664
30	0.1472	0.1671	0.1590	0.1410	0.1680	0.1552
50	0.1043	0.1214	0.1145	0.1036	0.1216	0.1130
75	0.0844	0.0972	0.0922	0.0851	0.0973	0.0913
100	0.0758	0.0863	0.0818	0.0755	0.0864	0.0813

Table 9

Estimates of Various Estimation Techniques for Parameter Lambda = 3.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	3.8912	3.7682	3.7442	3.4760	3.8282	3.7358
30	3.6443	3.6258	3.6155	3.4524	3.6471	3.6113
50	3.5539	3.5397	3.5356	3.4251	3.5524	3.5321
75	3.5210	3.5068	3.5058	3.4268	3.5153	3.5043
100	3.5301	3.5150	3.5158	3.4556	3.5214	3.5146

Table 10

Bias of Various Estimation Techniques for Parameter Lambda = 3.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.3913	0.2682	0.2442	-0.0240	0.3282	0.2358
30	0.1444	0.1258	0.1155	-0.0476	0.1471	0.1113
50	0.0539	0.0397	0.0356	-0.0749	0.0524	0.0321
75	0.0210	0.0068	0.0058	-0.0732	0.0153	0.0043
100	0.0302	0.0150	0.0158	-0.0444	0.0214	0.0146

Table 11

Mean Square Error of Various Estimation Techniques for Parameter Lambda = 3.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	1.7368	2.1464	1.9884	1.2697	2.2139	1.6920
30	0.4406	0.5924	0.5393	0.3787	0.6026	0.5012
50	0.2188	0.2974	0.2649	0.2067	0.3002	0.2559
75	0.1442	0.1885	0.1704	0.1420	0.1895	0.1665
100	0.1130	0.1460	0.1321	0.1094	0.1466	0.1298

Table 12

Mean Relative Estimates of Various Estimation Techniques Lambda = 3.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.2660	0.2908	0.2803	0.2417	0.2942	0.2664
30	0.1472	0.1671	0.1590	0.1410	0.1680	0.1552
50	0.1043	0.1214	0.1145	0.1033	0.1216	0.1130
75	0.0844	0.0972	0.0922	0.0848	0.0973	0.0913
100	0.0758	0.0863	0.0818	0.0751	0.0864	0.0813

Table 13

Estimates of Various Estimation Techniques for Parameter Lambda = 10.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	11.6738	11.3047	11.2326	10.4264	11.4846	11.2075
30	10.9331	10.8774	10.8466	10.3553	10.9412	10.8339
50	10.6617	10.6190	10.6068	10.2719	10.6571	10.5963
75	10.5631	10.5203	10.5175	10.2845	10.5460	10.5129
100	10.5905	10.5449	10.5475	10.3639	10.5643	10.5437

Table 14

Bias of Various Estimation Techniques for Parameter Lambda = 10.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	1.1738	0.8047	0.7326	-0.0736	0.9846	0.7075
30	0.4331	0.3774	0.3466	-0.1447	0.4412	0.3339
50	0.1617	0.1190	0.1068	-0.2281	0.1571	0.0963
75	0.0631	0.0203	0.0175	-0.2155	0.0460	0.0129
100	0.0905	0.0449	0.0475	-0.1361	0.0643	0.0437

Table 15

Mean Square Error of Various Estimation Techniques for Parameter Lambda = 10.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	15.6311	19.3177	17.8954	11.4254	19.9247	15.2282
30	3.9657	5.3318	4.8541	3.4048	5.4237	4.5110
50	1.9694	2.6769	2.3845	1.8552	2.7016	2.3034
75	1.2980	1.6963	1.5338	1.2707	1.7052	1.4988
100	1.0169	1.3137	1.1889	0.9748	1.3196	1.1683

Table 16

Mean Relative Estimates of Various Estimation Techniques Lambda = 10.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE
10	0.2660	0.2908	0.2803	0.2407	0.2942	0.2664
30	0.1473	0.1671	0.1590	0.1396	0.1680	0.1552
50	0.1044	0.1214	0.1145	0.1013	0.1216	0.1130
75	0.0844	0.0972	0.0922	0.0819	0.0973	0.0913
100	0.0758	0.0863	0.0818	0.0714	0.0864	0.0813

Table 17

Mean Square Error Ranking for Various Estimation Techniques for Fixed Parameter Values of Lambda = 0.5 and Lambda = 1.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE	MLE	LSE	WLSE	MPSE	CVME	ADE
10	3	5	4	1	6	2	3	5	4	1	6	2
30	2	5	4	1	6	3	2	5	4	1	6	3
50	2	5.5	4	1	5.5	3	2	5	4	1	6	3
70	1.5	5	4	1.5	6	3	2	5	4	1	6	3
100	2	5.5	4	1	5.5	3	2	5	4	1	6	3

Table 18

Mean Square Error Ranking for Various Estimation Techniques for Fixed Parameter Values of Lambda = 3.5 and Lambda = 10.5

n	MLE	LSE	WLSE	MPSE	CVME	ADE	MLE	LSE	WLSE	MPSE	CVME	ADE
10	3	5	4	1	6	2	3	5	4	1	6	2
30	2	5	4	1	6	3	2	5	4	1	6	3
50	2	5	4	1	6	3	2	5	4	1	6	3
70	2	5	4	1	6	3	2	5	4	1	6	3
100	2	5	4	1	6	3	2	5	4	1	6	3

Table 19

Best Parameter Estimation Techniques Based on the Simulation Studies Across All the Fixed Values of the Parameter

rank	10	30	50	75	100
1st	MPSE	MPSE	MPSE	MPSE	MPSE
2nd	MLE	MLE	MLE	MLE	MLE
3rd	ADE	ADE	ADE	ADE	ADE
4th	WLSE	WLSE	WLSE	WLSE	WLSE
5th	LSE	LSE	LSE	LSE	LSE
6th	CVME	CVME	CVME	CVME	CVME

DISCUSSION OF RESULTS

Tables 1-16 are illustrations of a simulation study conducted. The six techniques (MLE, ADE, CVME, MPSE, LSE, and WLSE) are explored in this article. On the other hand, Tables 1, 5, 9, and 13 reveal various estimates for the Sine-E parameters across the six techniques explored at different fixed values of the parameter. Estimations of estimation techniques approach the true value of the parameters as the sample size increases. Meanwhile, Tables 2, 6, 10, and 14 illustrate the biases of the different techniques explored at different fixed values of the parameter, and one can deduce that the biases reduce as the sample size increases. At the same time, Tables 3, 7, 11, and 15 illustrate the MSE. The MSE values decay as the sample sizes increase at different fixed values of the parameter. Notably, it is evidenced that the mean relative estimates of different estimation techniques decay as the sample sizes increase. This is portrayed in Tables 4, 8, 12, and 16. Accordingly, it is evident from the results that the six estimators possess consistency property.

CONCLUSION

Conclusively, all estimators' bias and MSE values decay as sample size increases, justifying improved accuracy in Sine-E distribution parameter estimation. As the parameter value increases, the bias and MSE increase, indicating lower precision at the high parameter value. However, as the parameter value decreases, the bias and the MSE decrease, indicating higher precision at lower parameter values. Nevertheless, the MPSE stands as the best estimator of the Sine-E distribution across various sample sizes and at the given parameter value, followed by MLE.

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