

# Design Verification of Heat Exchanger for Ballast Water Treatment

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## Article history

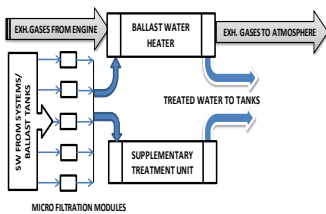
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## Graphical abstract



## Abstract

Using waste heat from ship's engines is one of the methods considered for heat treatment of ballast water. For such a system harvesting the engine exhaust heat, a heat exchanger will be vital. Design optimisation of a heater employing exhaust gases of the engine as utility fluid and ballast sea water as the process fluid was achieved using Lagrangian methods, keeping the annual cost as the objective function. Costs for installation, maintenance as also costs for the utility and process fluids were considered. Heat balance data, specific fuel consumption values from a typical operational ship and current fuel costs were considered for the design. The thermodynamic and geometric designs were worked out using computer based software for comparing the designs. Costs were also computed using a different approach for all the designs. Since the amount of heat transferred was specified and the application was limited to a single process, direct cost method was used for the computation. The objective function values obtained from Lagrangian equations were compared with the values obtained from direct cost computations. From the optimal designs, choice was justified based on annual cost, optimum exit temperature of shell side fluid and optimum mass flow of tube side fluid.

**Key words:** Ballast water treatment; waste heat recovery; heat exchanger; optimization; costs

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## 1.0 INTRODUCTION

International Convention for the Control and Management of Ships' Ballast Water and Sediments (BWM Convention) will come into force one year after the full ratification. As of 30<sup>th</sup> September 2013, 38 countries totalling 30.38% of global tonnage have agreed while the requirement for full ratification is 30 countries and 35% of world tonnage<sup>1</sup>. Subsequent to the ratification, ships have to comply with stricter performance standards. This is possible only by treatment of the ballast water replacing the current practice of ballast water exchange (BWE). Many ballast water treatment (BWT) systems are at commercial readiness after approvals but none of the systems are efficient enough to meet the stricter standards proposed by US Administration<sup>2</sup>. Research on BWT systems continues and heat treatment is one of the physical methods which have been probed into. Balaji and Yaakob<sup>3</sup> had shown heat availability on board though observing that heat treatment alone may not suffice for treating large quantities of ballast water. Though issues remain with heat treatment, the waste heat potential on board makes it an attractive option.

Most of available BWT systems are designed as a combination of two to five methods<sup>4,5</sup>. A ship board heat resource based system harvesting heat from engine exhaust gases in combination with another method could be a viable treatment system. Figure 1 shows a simple layout of such a BWT

combination system<sup>6</sup>. For this heat-reliant BWT system, a well-designed heat exchanger is essential.

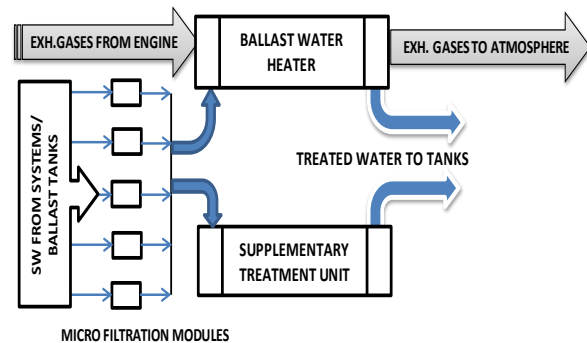


Figure 1 Ballast water treatment system

## 2.0 METHODOLOGY

Models have been proposed for optimising heat exchanger designs to enhance waste heat recoveries<sup>7</sup>. Heat exchanger design selection based on genetic algorithms<sup>8</sup> and multi objective optimisation<sup>9</sup> etc. have been proposed but these methods are suitable for processing plants involving a network of heat exchangers and other components. Since marine heat exchangers

are mostly singular, simpler optimisation techniques with engineer defined parameters and constraints can be employed.

## 2.1 Optimisation of Heat Exchanger

The heat duty for the heat exchanger was fixed considering a recovery of 10% from the input energy. A single pass, shell and tube heat exchanger having a counter flow pattern with baffles was designed. The fluids were assumed to undergo no phase change. Other assumptions included steady state operation, constant specific heats for the fluids, constant over all heat transfer coefficient and negligible heat losses<sup>10</sup>. The heat duty, inlet temperature of the shell side cold fluid and the tube diameters were treated as known variables.

### 2.1.1 Basic Equations for Optimisation

The objective function of annual cost can be written as an addition of annual variable costs, costs incurred for utility fluid and the power losses on the tube and shell circuits<sup>11,12</sup>.

$$C_T = A_o K_F C_{Ao} + m_u H_y C_u + A_o E_i H_y C_i + A_o E_o H_y C_o \quad (1)$$

$C_T$  is the total annual variable cost including operational costs.  $A_o$  ( $m^2$ ) is the area for heat transfer and  $K_F$  is the factor applied for computing fixed charges including maintenance (20% of the installation charges,  $C_{Ao}$ ).  $m_u$  (kg/s) is the mass flow of utility fluid,  $H_y$  denotes the hours of heat exchanger operation (taken as 2000 from vessel data) and  $C_u$  is the cost of utility fluid.  $E_i$  and  $E_o$  are the power losses incurred per unit area on the inside and outside of the tubes whereas  $C_i$  and  $C_o$  are the costs to pump the fluids.

The relationship for the thermal design is based on the enthalpy rate equations for single phase fluids where  $j = i, o$  denoting each of the fluids inside the tubes and outside respectively and  $\dot{m}_j$  representing the mass flow<sup>13</sup>.

$$q = q_j = \dot{m}_j \Delta h_j = (\dot{m}c_p)_j \Delta T_j = (\dot{m}c_p)_j [T_{j,i} - T_{j,o}] \quad (2)$$

For heat balance of hot and cold streams, the heat absorbed will be the product of the mass flows, specific heats and the temperature differences.

$$Q = m_c \cdot C_{pc} (t_2 - t_1) = m_h \cdot C_{ph} (T_1 - T_2) \quad (3)$$

Where  $m_c$  and  $m_h$  (kg/s) are the mass flow of cold (sea water) and hot fluid (exhaust gas), and  $C_{ph}$  and  $C_{pc}$  (J/kg K) are the specific heats of hot (exhaust gas) and cold fluid (sea water).

The inlet and outlet temperatures ( $^{\circ}C$ ) of the shell side fluid (sea water) are  $t_1$  and  $t_2$ . The inlet and outlet temperatures ( $^{\circ}C$ ) of tube side fluid are  $T_1$  and  $T_2$ .

The mass flow of the fluids can be obtained from,

$$m_u = \frac{Q}{C_{ph}(\Delta t_2 - \Delta t_1 + t_1 - t_2)} \quad (4a)$$

$$m_u = \frac{Q}{C_{pc}(\Delta t_1 - \Delta t_2 + T_1 - T_2)} \quad (4b)$$

Where  $\Delta t_1 = T_2 - t_1$  and  $\Delta t_2 = T_1 - t_2$  are the respective temperature differences between fluids in the counter flow pattern

at entry and exits. Equations (4a) and (4b) represent exhaust gases and sea water respectively.

The fundamental equation for heat transfer in the heat exchanger is,

$$Q = UA \Delta T_{lm} \quad (5)$$

$U$  ( $W/m^2 K$ ) is the Overall Heat Transfer Coefficient.  $A$  represents the surface area and  $\Delta T_{lm}$  ( $^{\circ}C$ ) is the Logarithmic Mean Temperature Difference (LMTD).

The optimum overall heat transfer coefficient can be calculated from,

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \cdot \frac{D_o}{D_i} + R_{fo} + R_{fi} \quad (6)$$

Where  $h_i$  and  $h_o$  ( $W/m^2 K$ ) are the Heat transfer coefficients for the inside and outside of the tubes respectively,  $D_i$  and  $D_o$  are the inside and outside diameters of the tube and  $R_{fi}$  and  $R_{fo}$  ( $m^2K/W$ ) are the fouling resistances on the tube and shell sides.

The overall heat transfer coefficient equation is further simplified by combining the fouling factors.  $R_{dw}$  ( $m^2K/W$ ) is the combined fouling resistance (tubes, scale & dirt).

$$U_o = \left( \frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right)^{-1} \quad (7)$$

The LMTD is calculated from,

$$\Delta T_{lm} = \frac{F(T_1 - t_2) - (T_2 - t_1)}{\ln \left[ \frac{T_1 - t_2}{T_2 - t_1} \right]} \quad (8)$$

The heat duty is then written as,

$$Q = F U_o A_o \frac{(\Delta t_2 - \Delta t_1)}{\ln(\Delta t_2 / \Delta t_1)} \quad (9)$$

The subscript 'o' represents the optimum value. A correction factor  $F$  is applied for counter current heat exchangers depending on the number of tube and shell passes of the process fluids. Equation (9) can be rewritten as,

$$\frac{1}{U_o A_o} = \frac{F(\Delta t_2 - \Delta t_1)}{Q \ln(\Delta t_2 / \Delta t_1)} \quad (10)$$

Substituting for  $U_o$  from equation (7),

$$\frac{F(\Delta t_2 - \Delta t_1)}{Q \ln(\Delta t_2 / \Delta t_1)} = \frac{1}{A_o} \left( \frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right) \quad (11)$$

The power losses inside and outside of the tubes,  $E_i$  and  $E_o$  are represented as follows<sup>11</sup>.  $\psi_i$  and  $\psi_o$  are the dimensional factors for estimating power losses in the tube and shell circuits.

$$E_i = \psi_i h_i^{3.5} \quad (12)$$

$$E_o = \psi_o h_o^{4.75} \quad (13)$$

These derivations were inserted in Equation (1) appropriately after substituting Equation (4) for  $m_u$  in (1). The objective function equation may be written for exhaust gases or sea water. Thus the objective function equations were structured on four variables  $\Delta t_2, A_o, h_i$  and  $h_o$  of which only three can be

independent. If three of the variables, say  $A_o$ ,  $h_i$  and  $h_o$  are known, the temperature difference  $\Delta t_2$  can be found.

### 2.1.2 Cost Computation using Lagrangian Multiplier

With the use of Lagrangian multipliers, optimal candidate points may be obtained where the problem is equality constrained<sup>14</sup>. With Equations (11), (12) and (13), the objective function can be expressed as an unconstrained problem with  $\lambda$ , the Lagrangian multiplier. Substituting Equations (4a), (4b), (11), (12) and (13), the Equation (1) can be expressed in the unconstrained form with the Lagrangian multiplier. Equations (14a) and (14b) represent exhaust gases and sea water respectively.  $C_{pu}$  is the specific heat of utility fluid (exhaust gas).

$$C_T = A_o K_F C_{Ao} + \frac{Q H_y C_u}{C_{pu} (\Delta t_2 - \Delta t_1 + t_1 - t_2)} + A_o \psi_i h_i^{3.5} H_y C_i + A_o \psi_o h_o^{4.75} H_y C_o + \lambda \left[ \frac{F(\Delta t_2 - \Delta t_1)}{Q \ln(\Delta t_2 / \Delta t_1)} - \frac{1}{A_o} \left( \frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right) \right] \quad (14a)$$

$$C_T = A_o K_F C_{Ao} + \frac{Q H_y C_u}{C_{pu} (\Delta t_1 - \Delta t_2 + T_1 - T_2)} + A_o \psi_i h_i^{3.5} H_y C_i + A_o \psi_o h_o^{4.75} H_y C_o + \lambda \left[ \frac{F(\Delta t_2 - \Delta t_1)}{Q \ln(\Delta t_2 / \Delta t_1)} - \frac{1}{A_o} \left( \frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right) \right] \quad (14b)$$

The obtained expressions are differentiable with respect to the four chosen variables resulting in following simultaneous equations. Equations (18a) and (18b) represent exhaust gases and sea water respectively. Solving the equations and eliminating  $\lambda$ , the optimum values can be obtained. The variables for optimum values are subscripted as 'opt'.

$$\frac{\partial C_T}{\partial h_i} = 3.5 A_o \text{opt} \psi_i h_i^{2.5} H_y C_i + \frac{\lambda D_o}{A_o \text{opt} D_i h_i^2 \text{opt}} = 0 \quad (15)$$

$$\frac{\partial C_T}{\partial h_o} = 4.75 A_o \text{opt} \psi_o h_o^{3.75} H_y C_o + \frac{\lambda D_o}{A_o \text{opt} D_i h_o^2 \text{opt}} = 0 \quad (16)$$

$$\frac{\partial C_T}{\partial A_o} = K_F C_{Ao} + \psi_i h_i^{3.5} H_y C_i + \psi_o h_o^{4.75} H_y C_o + \frac{\lambda}{A_o^2 \text{opt}} \left( \frac{D_o}{D_i h_i \text{opt}} + \frac{1}{h_o \text{opt}} + R_{dw} \right) = 0 \quad (17)$$

$$\frac{\partial C_T}{\partial \Delta t_2} = \left( \frac{\lambda F}{Q \ln(\Delta t_2 / \Delta t_1)} \right) + \left( \frac{F(\Delta t_1 - \Delta t_2)}{Q \Delta t_2 \ln(\Delta t_2 / \Delta t_1)^2} \right) + \frac{C_u H_y Q}{C_{pu} (\Delta t_1 - \Delta t_2 + t_1 - t_2)^2} = 0 \quad (18a)$$

$$\frac{\partial C_T}{\partial \Delta t_2} = \left( \frac{\lambda F}{Q \ln(\Delta t_2 / \Delta t_1)} \right) + \left( \frac{F(\Delta t_2 - \Delta t_1)}{Q \Delta t_2 \ln(\Delta t_2 / \Delta t_1)^2} \right) + \frac{C_u H_y Q}{C_{pu} (\Delta t_1 - \Delta t_2 + t_1 - t_2)^2} = 0 \quad (18b)$$

From equations (4a) and (4b) it can be seen that the mass flow of the utility fluid  $m_u$  depends on the temperature difference at the warm end  $\Delta t_2$ , while the other values are fixed. The

optimum value for the temperature difference at the warm end  $\Delta t_{2 \text{opt}}$  is obtained from the following equation.

$$\frac{F U_o \text{opt} H_y C_u}{C_{pu} (K_F C_{Ao} + E_i \text{opt} H_y C_i + E_o \text{opt} H_y C_o)} = \left( 1 + \frac{T_1 - T_2}{\Delta t_1 - \Delta t_2 \text{opt}} \right)^2 \left( \ln \frac{\Delta t_2 \text{opt}}{\Delta t_1} - 1 + \frac{\Delta t_1}{\Delta t_2 \text{opt}} \right) \quad (19)$$

The objective function values were computed for eight cases using the above equations. Four optimal design cases were developed with various cost considerations and for each design, optimum outlet temperature for sea water was calculated.

Three more cases were computed by keeping the outlet temperature of sea water constant but for similar cost considerations. One extra design was developed using software where Bell-Delaware approaches were employed with no cost considerations.

### 2.1.3 Computation of Direct Costs

The total annual costs will be an addition of capital cost  $C_{CA}$ , costs incurred in energy consumption  $C_E$ , and the operating costs  $C_s$ . Costs were computed for two considerations, one applying an interest rate on capital and a payback period and the other without considering both<sup>15</sup>.

$$C_{tot} = C_{CA} + C_E + C_s \quad (20)$$

Capital costs were computed by assuming two different values for energy costs. The first energy cost was obtained for ship board generation of unit power (US\$0.2/kWh) and the second one based on average local (ashore) cost (US\$0.06/kWh).

Capital Costs assuming these two energy costs were computed for two cases. In one case, costs for sea water pump (for process fluid) were considered and in another the pump costs were neglected. The cost of turbocharger was neglected as the cost would have been included with the main engine costs.

Standard values for all reference costs and indexes were obtained from relevant handbooks<sup>10,15</sup> as also from the Chemical Engineering Plant Cost Index (CEPCI) published by Chemical Engineering.

The capital cost was calculated from the following.

$$C_{CA} = \left( \frac{1}{n} + \frac{z}{2} \right) (I_{EX} + I_{P \text{tch}} + I_{P \text{pump}}) = a (I_{EX} + I_{P \text{tch}} + I_{P \text{pump}}) \quad (21)$$

$$I_{EX} = I_{EXo \text{ref}} \left( \frac{A_{EX}}{A_{EXo \text{ref}}} \right)^{m_{EX}} \quad (22)$$

$$I_{P \text{tch/pump}} = I_{P \text{tch/pump} \text{ref}} \left( \frac{L_{P \text{tch/pump}}}{L_{P \text{tch/pump} \text{ref}}} \right)^{m_P} \quad (23)$$

Where  $a$  the payback coefficient is equal to 0.125 and  $s$  is a coefficient based on intensity of maintenance. Assuming a medium intensity for cleaning and maintenance, a value of 0.025 was obtained from the Cost Index tables. The number of years of cost recovery period  $n$  was taken as 10 and the interest rate  $z$  was assumed to be 5%.

$I_{EX}$ ,  $I_{P\ tch}$  and  $I_{P\ pump}$  are the purchase costs of the heat exchanger, turbocharger and sea water pump computed from the reference costs  $I_{EX\ ref}$  and  $I_{P\ tch/pump\ ref}$  obtained from the industry indexes.

For the reference costs, the power (kW) referred to was denoted by  $L_{P\ tch/pump\ ref}$ . For the pump, the reference power was taken as 100 kW. The values of exponents  $m_{EX}$  and  $m_P$  were 0.59 and 0.67 as obtained from VDI Atlas<sup>15</sup>. The heat exchanger area and reference area are represented by  $A_{EX}$  and  $A_{EX\ ref}$ .

The energy costs and operating expenses were calculated as follows.

$$C_E = C_{EL} H_y \left( \frac{M_{ex\ gas} \Delta p_{tube}}{\rho_{eg} \eta_{tch}} + \frac{M_{pump} \Delta p_{shell}}{\rho_{sw} \eta_{pump}} \right) + C_{\Delta T} + C_M \quad (24)$$

$$C_s = s I_{EX} \quad (25)$$

The mass flow of exhaust gas  $M_{ex\ gas}$  was taken as 41.67 kg/s and sea water  $M_{pump}$  as 28.25 kg/s. The densities of the exhaust gas  $\rho_{eg}$  and sea water  $\rho_{sw}$  were 0.8767 kg/m<sup>3</sup> and 1017 kg/m<sup>3</sup> respectively. These values and the pressure drops  $\Delta p_{shell}$  and  $\Delta p_{tube}$  (Pa) were taken from the heat exchanger design parameters.

The sea water pump efficiency  $\eta_{pump}$  was taken as 0.7 and the turbocharger pump efficiency  $\eta_{tch}$  as unity. The additional energy costs for increasing temperature  $C_{\Delta T}$  and cost of supplies

$C_M$  were not considered as the heat exchanger was independent and not part of a network.

The objective function values and direct cost values were compared and variations observed. The least values of objective functions and the least variation from direct cost values were considered for selecting the area of the heat exchanger. The final design was determined by applying two other factors of mass flow and outlet temperature of sea water which are significant for the ballast water treatment.

### 3.0 RESULTS AND DISCUSSION

The assumed and derived costs used for calculations are tabulated in Table 1 and the cost summary in Table 2 shows the cost variations (in brackets). All designs were thermodynamically feasible. For treatment of ballast water, a minimum temperature of 55°C was kept as the target.

Case 1 was excluded because the optimum outlet temperature was much below the targeted temperature. Case 5 was used only as a reference for the geometrical values.

Case 2 had the highest objective function value followed by Case 3 and hence both were not considered, though other optimal values were tangible.

Amongst the rest of the cases, Case 8 had the least value of objective function, even lower than the direct costs. The next two cases with low objective function values were Case 7 and Case 4. The objective function value (Lagrangian) varied by 0.03% for Case 4 and 0.07% for Case 7, when compared with direct cost values.

But the obtained value of sea water outlet temperature (optimum value) was highest (>80 °C) for Case 4. This established the scope for higher mass flows and temperatures which are crucial for treating large volumes of ballast water<sup>3</sup>. So, Case 4 with least variation as also the highest optimum sea water

temperature was identified as the optimum design and all other geometric values were calculated for this design.

### 4.0 CONCLUSION

Aviable heat exchanger design employing simple optimisation techniques has been verified. Since cost was the chosen objective, another approach was employed to verify if the objective function values were closer and so the optimisation could be validated. For further validations, a heat exchanger model based on the chosen design has to be erected. The conditions assumed in the design have to be simulated to obtain the calculated temperatures. The species mortalities at these temperatures have to be assessed which will prove the effectiveness of the treatment method.

The scope of these discussions has been limited to cost computations only. The thermodynamic and geometric values used in developing the design and all equations related to heat exchanger design are not projected.

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**Table 1** Costs derived for design and verification

Costs (in US\$)	
Cost of purchase $C_{pur}$	123/m <sup>2</sup>
Cost of installation $C_{Ao}$	141.45/m <sup>2</sup>
Cost of utility fluid, exhaust gas $C_u$	0.04/kg
Cost to pump exhaust gas $C_i$	0.2/kWh
Cost to pump sea water $C_o$	0.2/kWh
Cost of energy ashore $C_{EL}$	0.06/kWh
Reference cost $I_{EXo ref}$	250/m <sup>2</sup>

**Table 2** Summary of cost computation (All costs in US\$)

	Area (m <sup>2</sup> )	Annual Cost	$I_{ppump}, C_{EL} = 0.2$	$I_{ppump} = 0, C_{EL} = 0.2$	$I_{ppump}, C_{EL} = 0.06$	$I_{ppump} = 0, C_{EL} = 0.06$	$I_{ppump}, C_{EL} = 0.2$	$I_{ppump} = 0, C_{EL} = 0.2$	$I_{ppump}, C_{EL} = 0.06$	$I_{ppump} = 0, C_{EL} = 0.06$
			Interest rate 5% payback 10 years				No Interest & payback			
<u><math>t_2</math> Optimum</u>										
Case 1 $C_u = 0, C_i = 0.2, C_o = 0.2$	363.32	14153.16	22167.63 (-0.57)	20292.63 (-0.43)	18833.99 (-0.33)	16958.99 (-0.20)	32704.25 (-1.31)	17704.25 (-0.25)	29370.61 (-1.08)	14370.61 (-0.02)
Case 2 $C_u = 0.04, C_i = 0.2, C_o = 0.2$	482.38	22124.65	24994.63 (-0.13)	23119.63 (-0.04)	21660.99 (0.02)	19785.99 (0.11)	35060.08 (-0.58)	20060.08 (0.09)	31726.45 (-0.43)	16726.45 (0.24)
Case 3 $C_u = 0.04, C_i = 0.2, C_o = 0$	474.65	22077.93	24820.50 (-0.12)	22945.50 (-0.04)	21486.86 (0.03)	19611.86 (0.11)	34914.97 (-0.58)	19914.97 (0.10)	31581.33 (-0.43)	16581.33 (0.25)
Case 4 $C_u = 0.04, C_i = 0, C_o = 0.2$	494.96	17477.46	25275.60 (-0.45)	23400.60 (-0.34)	21941.96 (-0.26)	20066.96 (-0.15)	35294.22 (-1.02)	20294.22 (-0.16)	31960.58 (-0.83)	16960.58 (0.03)
<u><math>t_2</math> fixed</u>										
Case 5 No costs; Software	383.70	n.a	22675.85	20800.85	19342.22	17467.22	33127.77	18127.77	29794.13	14794.13
Case 6 $C_u = 0.04, C_i = 0.2, C_o = 0.2$	435.61	20302.73	23922.63 (-0.18)	22047.63 (-0.09)	20588.99 (-0.01)	18713.99 (0.08)	34166.75 (-0.68)	19166.75 (0.06)	30833.11 (-0.52)	15833.11 (0.22)
Case 7 $C_u = 0, C_i = 0.2, C_o = 0$	428.56	16924.19	23757.02 (-0.40)	21882.02 (-0.29)	20423.39 (-0.21)	18548.39 (-0.10)	34028.74 (-1.01)	19028.74 (-0.12)	30695.11 (-0.81)	15695.11 (0.07)
Case 8 $C_u = 0, C_i = 0, C_o = 0.2$	434.90	12427.60	23906.00 (-0.92)	22031.00 (-0.77)	20572.36 (-0.66)	18697.36 (-0.50)	34152.89 (-1.75)	19152.89 (-0.54)	30819.25 (-1.48)	15819.25 (-0.27)