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Thermal Radiation Effect on Hydromagnetic Flow of Dusty Fluid over a Stretching Vertical Surface

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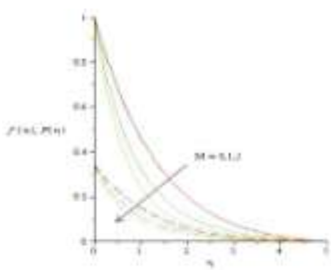
Graphical abstract



Abstract

In this paper, an analysis has been carried out to investigate the hydromagnetic fluid flow of dusty fluid with thermal radiation at vertical stretching sheet. The behavior of velocity and temperature profile of hydromagnetic fluid flow with fluid particle suspension is analyzed by using Runge Kutta Fehlberg forth-fifth order method (RKF45 Method). These solutions are presented and discussed for different parameters of interest such as fluid particle interaction parameter, the magnetic parameter, the radiation parameter, Grashof number, Eckert number and Prandtl number on the flow.

Keywords: Dusty fluid; two phase; magnetohydrodynamic; radiation; heat transfer

Abstrak

Dalam kertas kerja ini, analisis telah dijalankan untuk mengkaji aliran bendalir hidromagnet berdebu melalui satu lembaran regangan yang condong dengan kehadiran sinaran terma. Kelakuan profil halaju dan suhu aliran bendalir hidromagnet berdebu ini dianalisis menggunakan kaedah Runge-Kutta Fehlberg peringkat keempat-kelima (Kaedah RKF45). Penyelesaian ini dibincangkan bagi parameter yang berlainan kepentingan seperti parameter interaksi bendalir-zarah, parameter magnet, parameter radiasi, sudut kecondongan, nombor Eckert dan nombor Prandtl pada aliran.

Kata kunci: Bendalir berdebu; dua aliran fasa; magnetohidrodinamik; radiasi; pemindahan haba

1.0 INTRODUCTION

The study of fluid dynamics due to a stretching surface is essential due to its various applications to engineering and industrial disciplines. These applications include the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films. The quality of the resulting sheeting material depends considerably on the flow properties of the ambient fluid, speed of the collection and the rate of heat transfer at the stretching surface. Also the analysis of heat transfer with radiation is important in electrical power generation, solar power technology and other industrial fields. The properties of final product also depend highly on the rate of cooling. Therefore, the study of hydrodynamics flow also becomes important due to its applications in designing cooling systems with liquid metals and others materials processing. In view of these applications, Crane,[1] studied the two-dimensional boundary layer flow due to a stretching sheet. Then, several investigations have been done related to Crane,[1] flow problem with different physical situations have been reported.[2-5]

In the above investigations, they only deal the fluid flow induced by a linear stretching sheet without fluid particle suspension. The study of the boundary layer of fluid particle suspension is significant in determining the particle accumulation and impingement of the particle on the surface. Therefore, Chakrabarti,[6] investigated the boundary layer in a dusty gas. Then, Vajravelu and Nayfeh,[7] studied the boundary layer in a dusty fluid with the presence of magnetic flow. Recently, Gireesha et al.[8] studied the boundary layer flow and heat transfer of a dusty fluid over a stretching vertical surface.

Motivated by these analyses, this paper investigated the effect of thermal radiation and hydrodynamic flow on the dusty fluid over a stretching vertical surface. The coupled nonlinear partial differential equations governing the problem are transformed into a couple nonlinear ordinary differential equations by using similarity transformation. These nonlinear ordinary differential equations are solved numerically by using Runge Kutta Fehlberg forth-fifth order method (RKF45 Method) for different values of parameters of interest such as fluid particle interaction parameter, magnetic parameter, radiation parameter, Grashof number, Eckert number and Prandtl number on the flow

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2.0 MATHEMATICAL FORMULATION

Consider a steady two dimensional laminar boundary layer flow of a viscous, incompressible and electrically conducting dusty fluid over a vertical stretching sheet. The flow generated by action of two equal and opposite forces along the *x*-axis and *y*-axis being normal to the flow with stretching linear velocity $U_w(x)$ with prescribed surface temperature $T_w(x)$. The fluid and the dust particle clouds are supposed to be static at the beginning. Under these assumptions, the governing equations in the usual notation can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{KN}{\rho}\left(u_p-u\right)+g\beta^*\left(T-T_\infty\right)-\sigma_0 B_0^2 u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{N}{\rho\tau_T}\left(T_p-T\right)+\frac{N}{\rho c_p\tau_v}\left(u_p-u\right)^2-\frac{\partial q_r}{\partial y} \quad (3)$$

$$\frac{\partial}{\partial x}\left(Nu_p\right)+\frac{\partial}{\partial y}\left(Nv_p\right)=0 \quad (4)$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\frac{K}{m}\left(u-u_p\right) \quad (5)$$

$$u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}=\frac{K}{m}\left(v-v_p\right) \quad (6)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{c_p}{c_m\tau_T}\left(T_p-T\right) \quad (7)$$

where (u,v) and (u_p,v_p) denote the velocity components of the fluid and particle phases along the *x*-axes and *y*-axes respectively. υ is the coefficient of viscosity of the fluid, ρ and ρ_p are the density of the fluid and particle phase, B_0 is the induced magnetic field, β^* is the volumetric coefficient of thermal expansion, N is the number density of particle phase, K is the Stoke's resistance (drag co-efficient), m is the mass concentration of dust particles. T and T_p is the temperature of the fluid and temperature of the dust particles, c_p and c_m are specific heat of fluid and dust particles, τ_T is the thermal equilibrium time. τ_v is the relaxation time of the dust particles where the time required by the dust particles to adjust velocity relative to the fluid, k is the thermal conductivity and q_r is the radiative heat flux. In deriving these equations, the drag force is considered for the iteration between the fluid and particle phases. The boundary conditions are

$$u=U_w(x),\quad v=0,\quad T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2 \quad \text{at } y=0$$
$$u\to 0,\quad u_p\to 0,\quad v_p\to v, \quad (8)$$
$$N\to\rho\omega,\quad T\to T_\infty,\quad T_p\to T_\infty,\quad \text{as } y\to\infty$$

where $U_w(x)=bx$ is stretching linear velocity with prescribed surface temperature $T_w(x)=A(x/l)$ where $b(>)$ is stretching rate, l is characteristic length and A are constants. ω is the density ratio. By using Rosseland approximation for radiation,[9] radiation heat flux is simplified as

$$q_r=-\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (9)$$

where σ^* and k^* are Stefan-Boltzman constant and the mean absorption co-efficient, respectively. Assuming that temperature difference within the flow is such that T^4 may be expanded in Taylor's series. Expanding T^4 about T_∞ and neglecting higher order will obtain

$$T^4\approx 4T_\infty^3 T-3T_\infty^4 \quad (10)$$

Substituting Equation (8) and Equation (9) in Equation (6) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{N}{\rho\tau_T}\left(T_p-T\right)+\frac{N}{\rho c_p\tau_v}\left(u_p-u\right)^2+\left(k+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \quad (11)$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates in term of similarity variable and similarity function,[10] as

$$u=bxf'(\eta),\quad v=-\sqrt{\upsilon b}\,f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\eta=\sqrt{\frac{b}{\upsilon}}y,\ \rho_r=H(\eta),\quad u_p=bxF(\eta),\quad v_p=\sqrt{\upsilon b}G(\eta), \quad (12)$$
$$\theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty},\quad T-T_\infty=A\left(\frac{x}{l}\right)^2\theta(\eta).$$

By using the similarity equations from Equation (12), we obtain the following nonlinear ordinary differential equations:

$$f'''(\eta)+f(\eta)f''(\eta)-\left[f'(\eta)\right]^2+Gr\theta(\eta)+\beta H(\eta)\left[F(\eta)-f'(\eta)\right]-Mf(\eta)=0 \quad (13)$$

$$\left[1+\frac{4}{3}R\right]\theta''(\eta)+\Pr\left[f(\eta)\theta'(\eta)-2f'(\eta)\theta(\eta)\right]+\frac{\Pr N}{\rho\tau_T c}\left[\theta_p(\eta)-\theta(\eta)\right]+\frac{Ec\Pr N}{\tau_v\rho}\left[F(\eta)-f'(\eta)\right]^2=0 \quad (14)$$

$$H(\eta)F(\eta)+H(\eta)G'(\eta)+H'(\eta)G(\eta)=0 \quad (15)$$

$$G(\eta)F'(\eta)+\left[F(\eta)\right]^2+\beta\left[F(\eta)-f'(\eta)\right]=0 \quad (16)$$

$$G(\eta)G'(\eta)+\beta\left[f(\eta)+G(\eta)\right]=0 \quad (17)$$

$$2F(\eta)\theta_p(\eta)+G(\eta)\theta_p'(\eta)+\frac{c_p}{cc_m\tau_T}\left[\theta_p(\eta)-\theta(\eta)\right]=0 \quad (18)$$

where $\tau=mk$ is the relaxation time of the particle phase, $\beta=1/b\tau$ is the fluid particle interaction parameter, $Gr=g\beta^*\left(T_w-T_\infty\right)/b^2x$ is the Grashof number, $M=\sigma_0 B_0^2/\rho b$ is magnetic parameter and $\rho_r=N/\rho$ is relative density. While, $\Pr=\mu c_p/k$ is the Prandtl number, $Ec=cl^2/Ac_p$ is the Eckert number and $R=4\sigma^* T_\infty^3/kk^*$ is the radiation parameter.

The boundary condition in Equation (8) becomes

$$f(\eta)=0,\quad f'(\eta)=1,\quad \theta(\eta)=1 \ \text{at}\ \eta=0$$
$$f'(\eta)\to 0,\quad F(\eta)\to 0,\quad G(\eta)\to -f(\eta), \quad (19)$$
$$H(\eta)\to\omega,\quad \theta(\eta)\to 0,\quad \theta_p(\eta)\to 0 \ \text{as}\ \eta\to\infty.$$

3.0 RESULTS AND DISCUSSION

The system of coupled nonlinear ordinary differential equations as in Equations (13) to (18) with boundary condition Equation (19) is solved by using Runge-Kutta Fehlberg forth-fifth order method. The symbolic algebra software Maple is adopted given by Aziz[11] to solved these equations. Numerical solutions have been carried out to study the effect of various physical parameter such as fluid particle interaction parameter β , the magnetic parameter M , the radiation parameter R , Grashof number Gr , Eckret number Ec and Prandtl number $\Pr$ are shown graphically. In order to verify the accuracy of this study, the value of wall temperature $\theta'(0)$ gradient for different value of Prandtl number are given in Table 1. It shows the excellent agreement with reported by Grubka and Bobba[4] and Abel and Mahesha[5].

Table 1 Comparison the results for dimensionless temperature gradient $\theta'(0)$ in case of $\beta = N = Gr = M = R = 0$

Pr	Grubka and Bobba [4]	Abel and Mahesha [5]	Present Study
0.72	1.0885	1.0885	1.0916
1.0	1.3333	1.3333	1.3333
10.0	4.7969	4.7968	4.7964

Figure 1 shows the effect of magnetic parameter on the velocity profile for $\beta = N = 0.5, Ec = 2.0, \Pr = 1$ and $\omega = 0.2$. From this graph it is shown that the velocity of fluid and dust phase decreases as M increases. As M increase the Lorentz force which opposes the flow also increases and decreases the velocity of the flow. While, from Figure 2 shows the effect of Grashof number on velocity profile. It shows that the velocity of both fluid and dust phase increase as Gr increase. Physically, $Gr > 0$ means heating of the fluid or cooling the boundary surface and $Gr < 0$ means cooling of the fluid or heating boundary surface while $Gr = 0$ corresponds to the absence of free convection current.

the ratio of the two components has a profound effect on the microscopic structure and macroscopic properties of the gel in toluene. Figure 3 shows the effect of radiation parameter on temperature profile. It is observed that the increase in the thermal radiation parameter R produces significant increases in the thickness of the thermal boundary layer fluid. Figure 4 shows that the effect of fluid particle interaction parameter on temperature profile. We observed that temperature of the fluid and dust particle decrease with increase in β respectively. Figure 5 illustrates the effect of Eckert number on temperature profile of fluid and dust phase. It is evident from these graphs that increases in Ec increasing the temperature for both fluid and dust phase. Figure 6 is plotted for the temperature profile for different values of Prandtl number. We observed that the increasing number of Pr implies the decreasing temperature of fluid and dust phase.

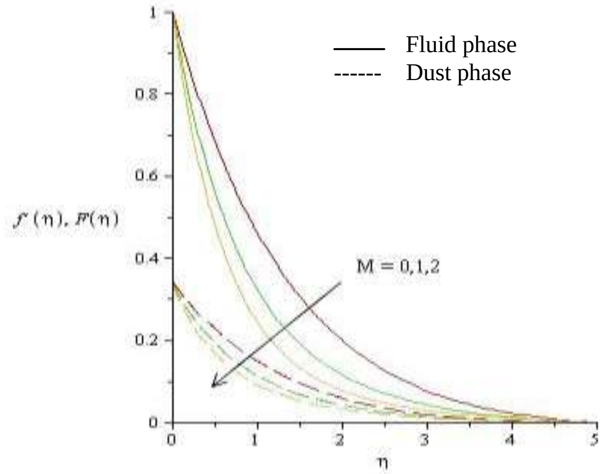

Figure 1 Effect of *M* on velocity profile.

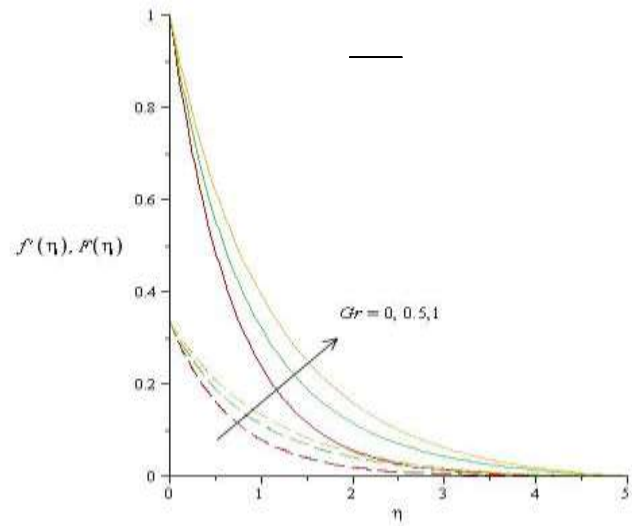

Figure 2 Effect of *Gr* on velocity profile.

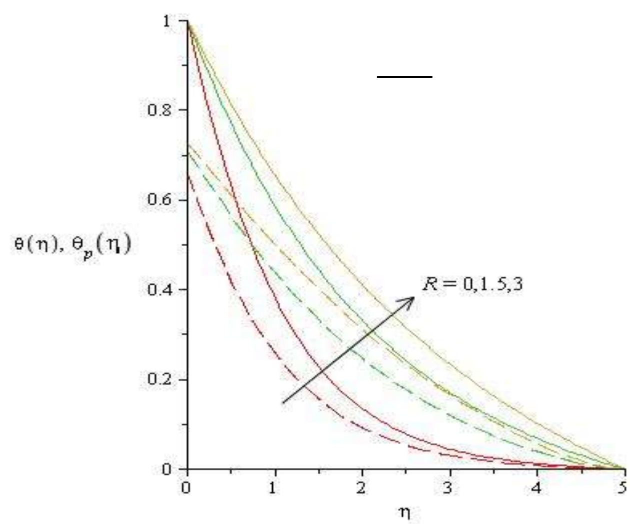

Figure 3 Effect of *R* on temperature profile.

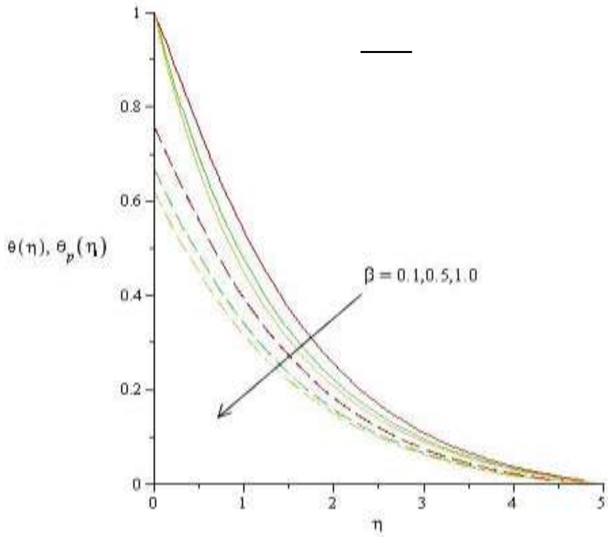

Figure 4 Effect of β on temperature profile.

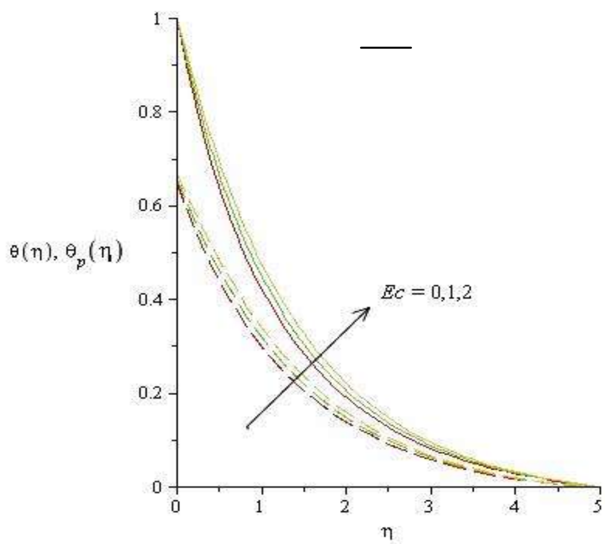

Figure 5 Effect of *Ec* on temperature profile.

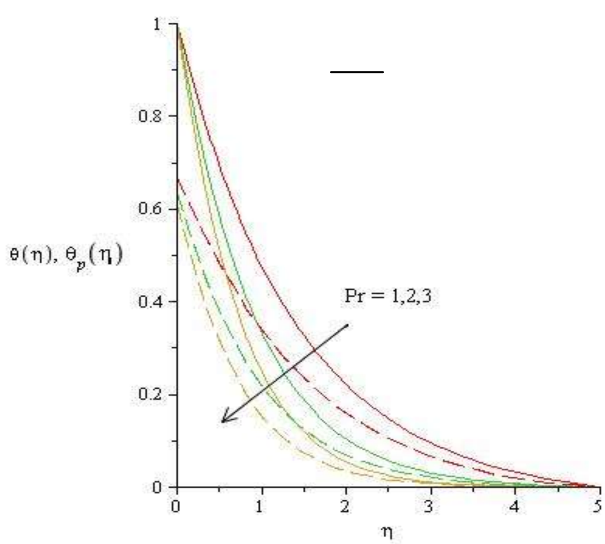

Figure 6 Effect of Pr on temperature profile.

■4.0 CONCLUSION

The hydromagnetic fluid flow and heat transfer of a dusty fluid with thermal radiation due to a vertical stretching sheet has been investigated. The effect of some parameters M, R, β, Gr, Ec and Pr controlling the velocity and temperature profiles are shown graphically and discussed briefly. Some of the important findings of our study are listed below:

- The effect of *M* is to decrease the momentum boundary layer thickness.
- The effect of *R* is to increase the thermal boundary layer thickness.
- The effect of *Gr* is to increase the momentum boundary layer.
- *Ec* increases the thermal boundary layer thickness.
- The boundary layers are highly influenced by the Prandtl number. The effect of Pr is to decrease the thermal boundary layer thickness.

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