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BACKSTEPPING CONTROL STRATEGY FOR AN UNDERACTUATED X4-AUV

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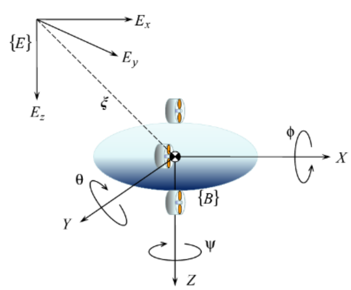
Graphical abstract



Abstract

A nonlinear control method is considered for stabilizing all attitudes and positions (x, y or z) of an underactuated X4-AUV with four thrusters and six degrees-of-freedom (DOFs), in which the positions are stabilized according to the Lyapunov stability theory and angles are stabilized using backstepping control method. A dynamical model is first derived, and then a sequential nonlinear control strategy is implemented for the X4-AUV, composed of translational and rotational subsystems. A controller for the translational subsystem stabilizes one position out of x-, y-, and z-coordinates, whereas controllers for the rotational subsystems generate the desired roll, pitch and yaw angles. Thus, the rotational controllers stabilize all the attitudes of the X4-AUV at a desired (x-, y- or z-) position of the vehicle. Some numerical simulations are conducted to demonstrate the effectiveness of the proposed controllers.

Keywords: Underactuated system, X4-AUV, backstepping control, Lyapunov stability theory

Abstrak

Kaedah kawalan tidak linear dipertimbangkan untuk menstabilkan pusingan dan kedudukan (x,y atau z) sebuah X4-Autonomous Underwater vehicle yang kurang penggerak dengan empat penujah dan enam darjah kebebasan (DOF), dimana kedudukan distabilkan berdasarkan kepada teori kestabilan Lyapunov dan sudut distabilkan dengan kaedah kawalan langkah kebelakang. Model dinamik diterbitkan dahulu, kemudian strategi kawalan sekuen tidak linear dilaksanakan untuk X4-AUV yang terdiri daripada peralihan dan putaran. Kawalan untuk subsistem peralihan menstabilkan satu kedudukan daripada koordinat x-,y- dan z-, manakala kawalan untuk subsistem putaran menjanakan sudut roll, pitch dan yaw yang dikehendaki. Oleh itu, pengawal putaran menstabilkan semua kedudukan X4-AUV pada kedudukan (x-, y- or z-) kenderaan tersebut. Simulasi berangka dijalankan untuk menunjukkan keberkesanan pengawal yang dicadangkan.

Kata kunci: Sistem kurang penggerak, X4-AUV, kawalan langkah belakang, teori kestbilan Lyapunov

1.0 INTRODUCTION

In the last few years, there has been major interest in developing stabilizing algorithms for underactuated systems. Underactuated systems are systems with fewer independent control actuators than degrees of freedom to be controlled. Hence, the dynamical equations of the vehicle exhibit so-called second-order nonholonomic constraints, i.e., non-integrable conditions imposed on the acceleration in one or more DOFs directions due to the lacks of capability to command instantanenous accelerations in these directions of the configuration space [1]. As pointed in a celebrated paper of Brockett in 1983 [2], such nonholonomic systems cannot be stabilized by usual smooth, time-invariant and state feedback control algorithms. Underactuation arises out of the need to reduce the actuators cost and weight or to increase the reliability of the system in case of actuator failure. The interest comes from the need to stabilize systems like ships, underwater vehicles, helicopters, aircraft, airships, hovercrafts, satellites, walking robots, etc., which may be underactuated by design.

Several control strategies based on passivity, Lyapunov theory, feedback linearization, etc. have been developed for the fully actuated case. However the techniques developed for fully actuated systems do not apply directly to the case of underactuated nonlinear systems.

An X4-AUV with an ellipsoid hull shape was studied by Zain [3][4], in which it makes only use of four thrusters to control the vehicle without using any steering rudders, falls into the class of underactuated AUVs since it has 6-DOFs (position (x, y, z), pitch, roll and yaw) and has nonholonomic features. The consideration of nonholomic systems is an interesting study from a theoretical standpoint, because as pointed out in the earlier works of Brockett, they cannot be asymptotically stabilized to a fixed point in the configuration space using continuously differentiable, time-invariant and state feedback control laws [2].

In this paper, we present a dynamic model of an underactuated X4-AUV with 6-DOFs and four control inputs and propose a control scheme based on the Lyapunov approach and backstepping control strategy to stabilize one position out of x-, y-, and z-coordinates and all the angles. The controller for the translational subsystem stabilizes the position and the controllers for the rotational subsystems generate the desired roll, pitch and yaw angles.

Chapters are organized as follows. In section 2, the coordinate system of an AUV is presented. The dynamic system of an X4-AUV is discussed in Section 3. Section 4, we present the control strategy to stabilize the X4-AUV. The discussion and simulation results are given in Section 5. Section 6 concludes the paper

2.0 DEFINITION OF COORDINATE SYSTEM

In order to describe the underwater vehicle's motion, a special reference frame must be established. There have two coordinate systems: i.e., inertial coordinate system (or fixed coordinate system) and motion coordinate system (or body-fixed coordinate system). The coordinate frame {E} is composed of the orthogonal axes {Ex Ey Ez} and is called as an inertial frame. This frame is commonly placed at a fixed place on Earth. The axes Ex and Ey form a horizontal plane and Ez has the direction of the gravity field. The body fixed frame {B} is composed of the orthonormal axes {X, Y, Z} and attached to the vehicle. The body axes, two of which coincide with principle axes of inertia of the vehicles, are defined in Fossen13 as follows:

X is the longitudinal axis (directed from aft to fore)
Y is the transverse axis (directed to starboard)
Z is the normal axis (directed from top to bottom)

Figure 1 shows the coordinate systems of AUV, which consist of a right-hand inertial frame {E} in which the downward vertical direction is to be positive and right-hand body frame {B}.

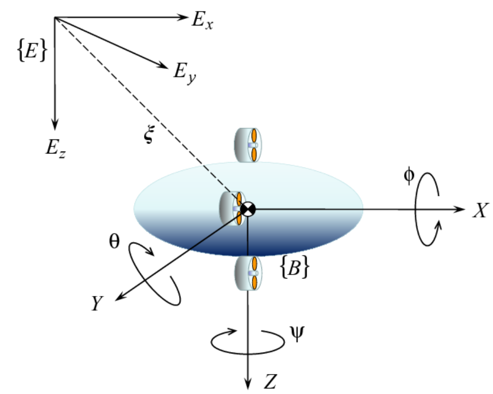


Figure 1 Coordinate systems of AUV

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the mass center of the body in the inertial frame, defining the rotational angles of X-, Y- and Z-axis as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$, the rotational matrix R from the body frame {B} to the inertial frame {E} can be reduced to:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

where cα denotes cos α and sα is sin α.

3.0 SYSTEM DESCRIPTION

Defining $\boldsymbol{q} = [\xi^T \ \eta^T]$, the dynamical model of an X4-AUV is described in the following matrix form:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + V_m(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = B(\boldsymbol{q})\boldsymbol{\tau} \quad (2)$$

where $M(\boldsymbol{q}) \in \Re^{6\times6}$ is the symmetric, positive definite inertia matrix, $V_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^{6\times6}$ is the centrifugal and Coriolis matrix, $G(\boldsymbol{q}) \in \Re^6$ is the gravitational vector, $B(\boldsymbol{q}) \in \Re^{6\times4}$ is the input transformation matrix, and $\boldsymbol{\tau} \in \Re^4$ is a generalized force vector consisting of force or torque components.
Note also that each matrix in the dynamical model can be reduced to

$$M(\boldsymbol{q}) = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}$$

$$B(\boldsymbol{q}) = \begin{bmatrix} c\theta c\psi & 0 & 0 & 0 \\ c\theta s\psi & 0 & 0 & 0 \\ -s\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & l & 0 \\ 0 & 0 & 0 & l \end{bmatrix}$$

$$V_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_z\dot{\psi} & -I_y\dot{\theta} \\ 0 & 0 & 0 & -I_z\dot{\psi} & 0 & J_t\Omega + I_x\dot{\phi} \\ 0 & 0 & 0 & I_y\dot{\theta} & -J_t\Omega - I_x\dot{\phi} & 0 \end{bmatrix}$$

$$\text{diag}(m_1, m_2, m_3) = m_b I + M_f$$
$$\text{diag}(I_x, I_y, I_z) = J_b + J_f$$

Here, m_1, m_2 and m_3 is a total mass in the x-, y- and z-direction, I_x, I_y and I_z is a total inertia in the x-, y- and z- direction, J_t is a total thruster inertia, l is a horizontal distance from the propeller center to the center of gravity, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle, I denotes the unit matrix, M_f is an added mass matrix, and J_f is an added moment of inertia matrix. Assuming that the fluid density is ρ and the present AUV form is ellipsoid, it is found that suitable M_f and J_f are obtained.[15] Furthermore assume that the X4-AUV is in the state of neutral buoyancy to neglect the potential energy, so that $G(\boldsymbol{q}) = 0$. From the rotational matrix (1), the kinematic equation for X4-AUV.

$$\dot{\boldsymbol{q}} = S(\boldsymbol{q})\boldsymbol{v} \quad (3)$$

can be reduced to

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c\theta c\psi & 0 & 0 & 0 \\ c\theta s\psi & 0 & 0 & 0 \\ -s\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_b \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (4)$$

because the lateral type X4-AUV has only the total thrust in the X-direction, where $v = [\dot{x}_b \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$, where $\dot{x}_b$ denotes the X-directional translational velocity and $[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$ is the rotational angular velocity vector in the body frame.

Therefore, the dynamic equations of motion for an X4-AUV in Equation (2) can be written as

$$\begin{aligned} m_1\ddot{x} &= \cos\theta\cos\psi u_1 \\ m_2\ddot{y} &= \cos\theta\sin\psi u_1 \\ m_3\ddot{z} &= -\sin\theta u_1 \\ I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_z) + u_2 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) - J_t\dot{\theta}\Omega + lu_4 \end{aligned} \quad (5)$$

The whole system is setup by connecting the PI camera module to the CSI port on the Raspberry PI board via ribbon cable while the LCD screen is connected to the board via HDMI cable. The wireless keyboard and mouse is connected to the board using wireless USB adapter. This is only needed when manipulation of code is required. The power is supplied to the board by connecting a micro USB to USB cable to a wall socket USB adapter or power bank.

4.0 CONTROL STRATEGY OF AN X4-AUV

The model (5), can be rewritten in a state-space form $\dot{X} = f(X, U)$ by introducing $X = (x_1 \cdots x_{12})^T \in \Re^{12}$ as state vector of the system as follows:

$$\begin{array}{l|l} x_1 = x & x_7 = \emptyset \\ x_2 = \dot{x}_1 = \dot{x} & x_8 = \dot{x}_7 = \dot{\phi} \\ x_3 = y & x_9 = \theta \\ x_4 = \dot{x}_3 = \dot{y} & x_{10} = \dot{x}_9 = \dot{\theta} \\ x_5 = z & x_{11} = \psi \\ x_6 = \dot{x}_5 = \dot{z} & x_{12} = \dot{x}_{11} = \dot{\psi} \end{array} \quad (6)$$

where the inputs $U = (u_1 \cdots u_2)^T \epsilon \Re^4$.

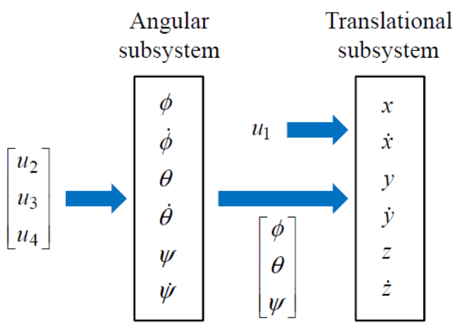


Figure 2 Connection of rotational and translational subsystems

From (5) and (6), we obtain:

$$f(X,U) = \begin{pmatrix} x_2 \\ (\cos\theta\ \cos\psi)\dfrac{1}{m_1}u_1 \\ x_4 \\ (\cos\theta\ \sin\psi)\dfrac{1}{m_2}u_1 \\ x_6 \\ (-\sin\theta)\dfrac{1}{m_3}u_1 \\ x_8 \\ x_{10}x_{12}\left(\dfrac{\mathrm{I_y}-\mathrm{I_z}}{\mathrm{I_x}}\right) + \dfrac{l}{\mathrm{I_x}}u_2 \\ x_{10} \\ x_8x_{12}\left(\dfrac{\mathrm{I_z}-\mathrm{I_x}}{\mathrm{I_y}}\right) - \dfrac{\mathrm{J_t}}{\mathrm{I_y}}x_{12}\Omega + \dfrac{l}{\mathrm{I_y}}u_3 \\ x_{12} \\ x_8x_{10}\left(\dfrac{\mathrm{I_x}-\mathrm{I_y}}{\mathrm{I_z}}\right) + \dfrac{\mathrm{J_t}}{\mathrm{I_z}}x_{10}\Omega + \dfrac{l}{\mathrm{I_z}}u_4 \end{pmatrix} \quad (7)$$

with:

$$\begin{array}{l|l} a_1 = (\mathrm{I_y} - \mathrm{I_z})/\mathrm{I_x} & b_1 = 1/\,\mathrm{I_x} \\ a_2 = (\mathrm{I_z} - \mathrm{I_x})/\mathrm{I_y} & b_2 = l/\,\mathrm{I_y} \\ a_3 = \mathrm{J_t}/\mathrm{I_y} & b_3 = l/\,\mathrm{I_z} \\ a_4 = \mathrm{J_t}/\mathrm{I_z} & \\ a_5 = (\mathrm{I_x} - \mathrm{I_y})/\mathrm{I_z} & \end{array}$$

It is worthwhile to note in the latter system that the angles and their time derivatives do not depend on translation components. On the other hand, the translations depend on the angles. We can ideally imagine the overall system described by (7) as constituted of two subsystems, the angular rotations and the linear translations, see Figure 2.

4.1 Control of the Rotations Subsystem

Using the backstepping approach, one can synthesize the control law forcing the system to follow the desired trajectory. For the first step we consider the tracking-error:

$$z_1 = x_{7d} - x_7 \quad (8)$$

And we use the Lyapunov theorem by considering the Lyapunov function z_1 positive definite and it's time derivative negative semi-definite:

$$\mathrm{V}\,(z_1) = \tfrac{1}{2}{z_1}^2 \quad (9)$$

$$\dot{\mathrm{V}}(z_1) = z_1(\dot{x}_{7d} - x_8) \quad (10)$$

The stabilization of z_1 can be obtained by introducing a virtual control inputx_8:

$$x_8 = \dot{x}_{7d} + \alpha_1 z_1 \text{with: } \alpha_1 > 0 \quad (11)$$

The Equation (6) is then:

$$\dot{\mathrm{V}}(z_1) = -\alpha_1 {z_1}^2 \quad (12)$$

Let us proceed to a variable change by making:

$$z_2 = x_8 - \dot{x}_{7d} - \alpha_1 z_1 \quad (13)$$

For the second step we consider the augmented Lyapunov function:

$$V(z_1, z_2) = \frac{1}{2}{z_1}^2 + \frac{1}{2}{z_2}^2 \quad (14)$$

And it's time derivative is then:

$$\begin{aligned} \dot{\mathrm{V}}\,(z_1, z_2) = &\ z_2(\,a_1x_{10}x_{12} + b_1u_2) \\ &- z_2(\ddot{x}_{7\mathrm{d}} - \alpha_1(z_2 + \alpha_1 z_1\,)\,) \\ &- z_1z_2 - \alpha_1{z_1}^2 \end{aligned} \quad (15)$$

The control input u_2 is then extracted $(\ddot{x}_{1,2,3\mathrm{d}} = 0)$, satisfying $\dot{\mathrm{V}}\,(z_1z_2) < 0$:

$$u_2 = \frac{1}{b_1}(z_1 - a_1x_{10}x_{12} - \alpha_1(z_2 + \alpha_1 z_1\,) - \alpha_2 z_2) \quad (16)$$

The term $\alpha_2 z_2$ with $\alpha_2 > 0$ is added to stabilize z_1. the same steps are followed to extract u_3 and u_4

$$\begin{aligned} u_3 = \frac{1}{b_2}((z_3 - &\ a_2x_8x_{12} - a_3x_{12}\Omega) \\ &- \alpha_3(z_4 + \alpha_3 z_3\,) - \alpha_4 z_4) \end{aligned} \quad (17)$$

$$\begin{aligned} u_4 = \frac{1}{b_3}((z_5 - &\ a_5x_8x_{10} - a_4x_{10}\Omega) \\ &- \alpha_5(z_6 + \alpha_5 z_5) - \alpha_6 z_6) \end{aligned} \quad (18)$$

with:

$$\begin{cases} z_3 = x_{9d} - x_9 \\ z_4 = x_{10} - \dot{x}_{9d} - \alpha_3 z_3 \\ z_5 = x_{11d} - x_{11} \\ z_6 = x_{12} - \dot{x}_{11d} - \alpha_5 z_5 \end{cases} \quad (19)$$

Note that this technique also used for a Quadrotor studied in [5].

4.2 Control of the Linear Translations Subsystem

Let us consider the simple task for the X4-AUV to be translated to a particular position $x = x^d$. The dynamics of the position are described by lines 1 and 2 of system (3), i.e.,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ (\cos x_9 \ \cos x_{11}) \dfrac{u_1}{m_1} \end{pmatrix} \quad (20)$$

By the considerations in the control for the subsystem of the angular rotations in [3], we ensure that starting from an initial condition where V(Xα)< $\pi/2$ the angles and their velocities are constrained in this hypersphere of$\Re^6$. In this case $\cos x_9 \cos x_{11} \neq 0$ during all the trajectories of the system under the previous control law. If the latter condition is satisfied, we can linearize system (20) by simply compensating the weighted force by:

$$u_1 = \frac{m_1 \hat{u}_1}{\cos x_9 \ \cos x_{11}} \quad (21)$$

where $\hat{u}_1$ is an additional term. By this partial feedback linearization, (16) becomes:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ \hat{u}_1 \end{pmatrix} \quad (22)$$

or

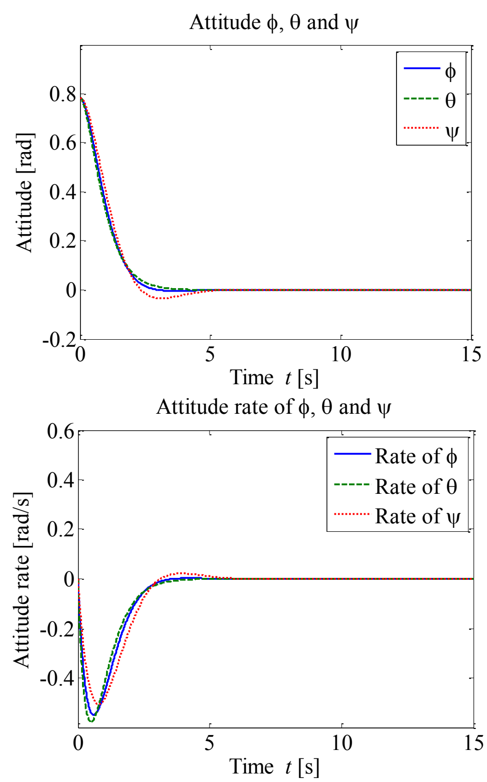
$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} e_2 \\ -\hat{u}_1 \end{pmatrix} \quad (23)$$

in the error form, where $e_i \triangleq x_i^d - x_i, i = 1,2$. Adopting a simple linear state feedback stabilization law $\hat{u}_1 = k_1 e_1 + k_2 e_2$, we can stabilize the position by placing the poles of the subsystem in any position in the complex left half plane.

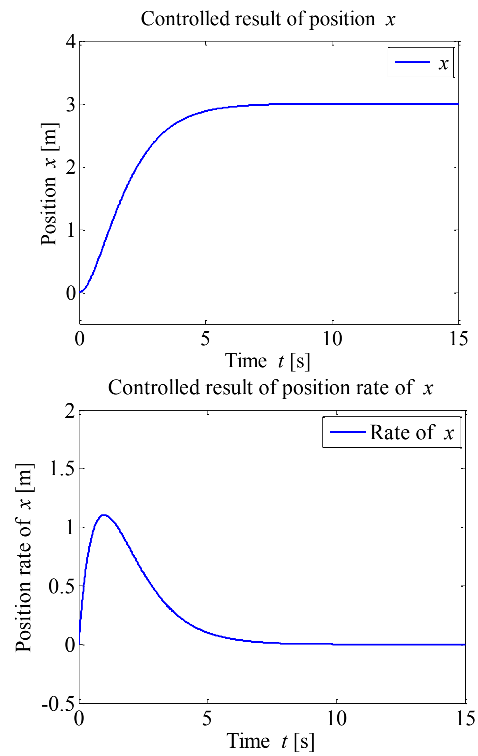
5.0 DISCUSSIONS

A nonlinear control strategy is implemented to stabilize the X4-AUV. The position and angles of the X4-AUV are stabilized by using control input u_1, u_2, u_3, and u_4 respectively. Backstepping controllers were introduced for controlling each orientation angle and the positions are stabilized according to the Lyapunov stability theory. The angles and their time derivatives of rotational subsystem do not depend on translation components, whereas the translations depend on the angles. Ideally, it can be imagined as two subsystems: the angular rotations and the linear translations. Due to its complete independence from the other subsystem, the angular rotation-related subsystem is tuned first. The rotational control keeps a 3D orientation of the X4-AUV to the desired state and the translational control moves the vehicle to the desired position.

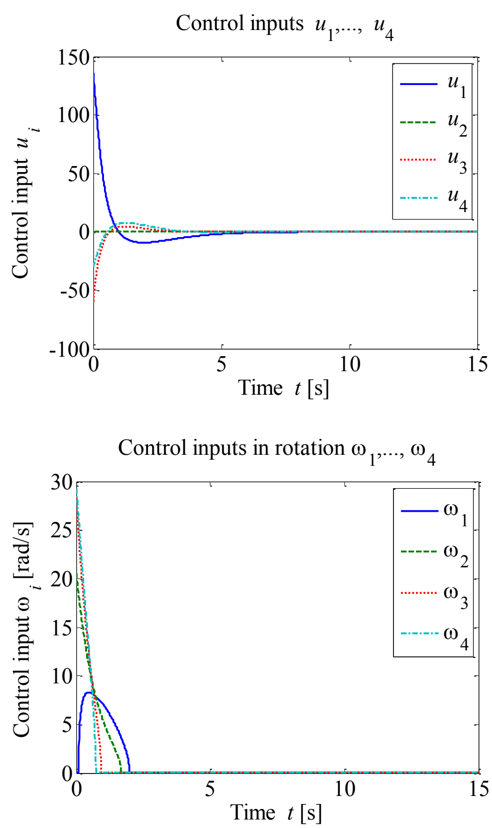
The controllers have been implemented on MATLAB and the simulation results for stabilizing the X4-AUV in x-positions are shown in Figure 3. The system started with an initial state $X_0 = (0,0,0,0,0,0,\frac{\pi}{4},0,\frac{\pi}{4},0,\frac{\pi}{4},0)^T$ and we wanted the final x-positions, at 3 m with all zero orientation angles. As shown in Figure 2, it is seen that all orientation angles, and x-positions converge to the targets, where α_1 = 1, α_2 = 2, α_3 = 3, α_1 = 1, α_4 = 1, α_5 = 1, α_6 = 1, k_1 = 1.0, K_2 = 2.0. The physical parameters for X4-AUV that has been used for simulating the dynamic model presented in Table 1. Note that the simulations for stabilizing the X4-AUV in x-, y- and z-positions were implemented independently. The other results for y- and z-position are not included in this paper.



(a) Attitude and attitude rate control for x-position



(b) A Position and position rate control for x-position



(c) A control inputs and control inputs in rotation

Figure 3 A case for stabilizing the orientation angles and x-axis position

Table 1 Physical parameters for X4-AUV

Parameter	Description	Value	Unit
m_b	Mass	21.43	Kg
ρ	Fluid density	1023.0	kg/m^3
l	Distance	0.1	M
r	Radius	0.1	m
b	Thrust factor	0.068	N·s^2
d	Drag factor	3.617e^{-4}	N·m·s^{-2}
J_{bx}	Roll inertia	0.0857	kg·m^2
J_{by}	Pitch inertia	1.1143	kg·m^2
J_{bz}	Yaw inertia	1.1143	kg·m^2
J_t	Thrust inertia	1.1941e^{-4}	N·m·s^{-2}

6.0 CONCLUSIONS

In this paper, a nonholonomic control method has been presented for stabilizing all attitudes and positions (*x*, *y* or *z*) of an underactuated X4-AUV with four thrusters and 6-DOFs. The controller design was separated into two parts: the rotational and translational dynamics-related control designs. The stabilization strategy is based on the Lyapunov stability theory and backstepping control method. For the future work, an underactuated controller will be constructed by combining such three types of controllers to realize an underactuated control system.

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