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A Three Competing Species Model for Wastewater Treatment: Case Study on Taman Timur Oxidation Pond, Johor Bahru

Amir S. A. Hamzah[a,b], Ali H. M. Murid[a,b,*], Zaitul Marlizawati Zainuddin[a,b], Zainal A. Aziz[a,b], Hasniza Ramli[c], Hazzarita Rahman[c], Norazelah Hamdon[c]

[a]UTM Centre for Industrial and Applied Mathematics Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[b]Department of Mathematical Sciences, Faculty of Science Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[c]Johor Biotechnology & Biodiversity Corporation (J-Biotech), Level 2, UTM-MTDC Technology Centre, Phase2 Technovation Park, 81300 Skudai, Johor, Malaysia

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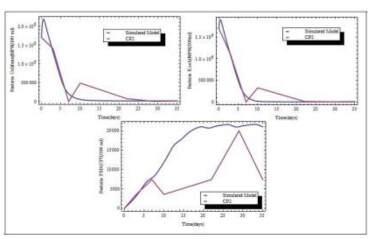
*Corresponding author
alihassan@utm.my

Graphical abstract



Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic)

Abstract

Oxidation pond techniques have practically proved to be effective for wastewater treatment process (WWTP) because of their low construction and operating cost. Cumbersome sampling is required to monitor the dynamics of the WWTP which also involves enormous costly work. Deterministic model accommodating the correlation between the amount of phototrophic bacteria in a product called mPHO (bioproduct used to improve water quality) and pollutant (bacteria E.coli and Coliform) existing in oxidation pond is developed to facilitate the analysis of this process. This study presents ordinary differential equation model for an oxidation pond to investigate the effect of mPHO on the degradation of pollutant. The model consists of a system of ordinary differential equations (ODE) with coupled reaction equations for the pollutant and phototrophic bacteria. The parameters of the model is estimated using the real data collected from an aerobic oxidation pond located in Taman Timor, Johor, Malaysia to illustrate a real life application of this model. The simulation results provide a better understanding of the ecological system in the oxidation pond.

Keywords: Mathematical modeling, ordinary differential equation, wastewater treatment process (wwtp)

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1.0 INTRODUCTION

A mathematical model is a simplified version of the real world process that employs the tools of mathematics like statistics, probability theory, graph theory, and differential equations. These mathematical methods help us to understand the nature of the problem that cannot be clearly interpreted. Sometimes to get a solution for the current problems, one needs to develop a new method to solve the problem or modify the standard methods that have been successful in the past. These challenges should be taken as a motivation for researchers to model the current problems mathematically to facilitate it to be understood and explained to the public.
A real wellspring of water contamination originates from sewage, and it could be a genuine issue when the municipal wastewater is not fittingly treated. Innovative

treatment has both generally high capital and operational expenses. An option to this problem is to create and apply the advances with lower expenses and better execution. Oxidation pond techniques have become very popular with small communities because of their low construction and operating cost, as well as a significant financial advantage over other recognized treatment methods [1]. Oxidation ponds are stages in a water treatment plant in which solid effluent and organic matter are degraded either in anaerobic or aerobic system.

The wastewater treatment plant that becomes our pilot study is located at Taman Timor oxidation pond, Johor Bahru, estimated about 1,909 square metres and about 1.5 metres in depth, 54 metres length and about 2,864.13 cubic metres of total volume of water. In order to intensify the effectiveness of oxidation pond technique and accelerate the population of phototrophic bacteria in the pond, mPHO (biological based product) that contains mainly phototrophic bacteria has been added regularly within three months period of study. Samples were collected at two points, which are CP1 (influent and application of mPHO) and CP2 (effluent) (Refer to Figure 3).

2.0 LITERATURE REVIEW

Several types of mathematical modelling have been widely used in biology. These models play an important role in assisting various fields such as medical, ecological, and plantation. Such models provide better understanding in unvisualised phenomena, for instance, the interaction between microorganisms (bacteria), blood flow through vein, and the cycle of photo synthesis. In order to fit these needs, many researchers have made significant contributions in developing models that can provide solutions to all problems.

Numerous models that can predict the behaviour of ecological system have been described by both deterministic and stochastic approaches [2]. Among the earliest models that were developed to describe the interaction between populations have been described using ordinary differential equations. The model was known as the Lotka-Volterra or the predator-prey model. The model was named after Alfred J. Lotka an American mathematician and Vito Volterra an Italian mathematician. The most basic of the model is related to a system of two ordinary differential equations, the first related to predator and second for it's prey. Interdependent relations from predator and it's prey are required to stabilize an ecosystem.

At the very beginning, these deterministic models seem to be fitted in the condition that we want to investigate in ecology. Thsese models have been through the improvement with so many results obtained based on the original model of Lotka-Volterra. There are also studies that have been made on this predator-prey model with the effects of disease in the system, either to the predator or it's prey [3-5]. A review on significant contributions on these models can be found on [6,7]. These models were further improved using the advancements in computing, by considering the dynamical structure of biological system.

3.0 MATHEMATICAL MODEL

This section provides a description of mathematical model that have been developed to study the biological wastewater treatment process that occur in oxidation pond. Location for application of mPHO and sampling points are as presented in Figure 1. From the first week, 25 liters of mPHO were applied each day with 2 days break in the weekend followed by 30 liters of mPHO each day for the second week. This phase is considered as the first phase of application. For the third week until fifth week 10 liters of mPHO were applied each day with also 2 days break in the weekend (refer to Table 1 and Table 2). From Phase 1 and Phase 2 we have a continuous treatment of mPHO application within 35 days as illustrated in Figure 2. Figure 3 shows the graphs of bacteria (E.coli and Coliform) based on the data collected at CP1 within 35 days.

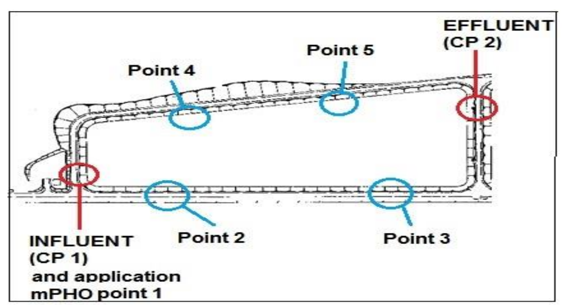

Figure 1 Sampling and application location at taman timor oxidation pond

Table 1 Treatment schedule of mpho application at taman timor oxidation pond (phase 1)

Treatment Approaches	Day (Date)	Application / Activities	Quantity /Frequency	Location
		Pre- Sampling	3 times	Both Point (CP 1 & CP 2)
Intensive (Phase 1)		**Sampling**	**2 locations**	**Both Point (CP 1 & CP 2)**
	1 (14/11/2014)	mPHO	25 liters	CP 1 (Influent)
	2 (15/11/2014)	mPHO	25 liters	CP 1 (Influent)
	3 (16/11/2014)	mPHO	25 liters	CP 1 (Influent)
		Sampling	2 locations	Both Point (CP 1 & CP 2)
	4 (17/11/2014)	mPHO	25 liters	CP 1 (Influent)
	5 (18/11/2014)	mPHO	25 liters	CP 1 (Influent)
	(19-20/11/2014)	**Break for 2 days**		
		Sampling	**2 locations**	**Both Point (CP 1 & CP 2)**
	8 (21/11/2014)	mPHO	30 liters	CP 1 (Influent)
	9 (22/11/2014)	mPHO	30 liters	CP 1 (Influent)
	10 (23/11/2014)	mPHO	30 liters	CP 1 (Influent)
		Sampling	2 locations	Both Point (CP 1 & CP 2)
	11 (24/11/2014)	mPHO	30 liters	CP 1 (Influent)
	12 (25/11/2014)	mPHO	30 liters	CP 1 (Influent)

Table 2 Treatment schedule of mpho application at taman timor oxidation pond (phase 2)

Treatment Approaches	Cycles (week)	Application / Activities	Quantity (Liter)	Location
Continuous (Phase 2)	1^{st} (28/11/2014- 4/12/2014)	Application mPHO	100	CP 1 (Influent)
	2^{nd} (5/12/2014- 11/12/2014)	**Sampling**	1	CP 1 & CP 2
		Application mPHO	100	CP 1 (Influent)
	3^{rd} (12/12/2014- 18/12/2014)	Application mPHO	100	CP 1 (Influent)

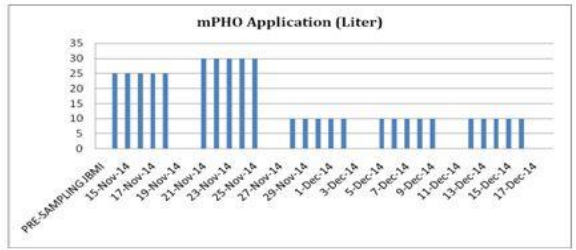

Figure 2 Treatment schedule of mpho application at taman timor oxidation pond

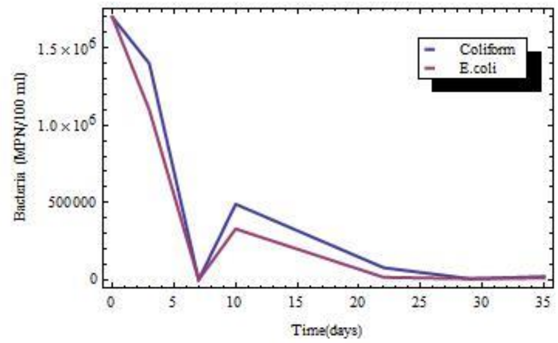

Figure 3 Graph of bacteria (e.coli and coliform) at cp1 within 35 days

Mathematical Model: Three competing species

We model the wastewater treatment process based on ordinary differential equation (ODE) model with interaction that occurs between microorganism (bacteria E.coli, bacteria Coliform and Phototrophic) [8,9].

The variables and parameters used in this model are as follows:

The volume of the oxidation pond initially contains 2864125.13 liter of wastewater.
The mixtures are kept uniform by stirring.
G^* is the number of phototrophic bacteria in mPHO (1.91×10^9 CFU/100 ml).
BE^* is the number of bacteria E.coli coming from outside (1.6×10^7 MPN/100 ml).
BC^* is the number of bacteria Coliform coming from outside (1.6×10^7 MPN/100 ml).
$x_1(t)$ is the number of phototrophic bacteria in the pond (CFU/100 ml) where t varies from initial time up to 35 days.
$x_2(t)$ is the number of E.coli in the pond (MPN/100 ml).
$x_3(t)$ is the number of Coliform in the pond (MPN/100 ml).
$x_p(t)$ is the sewage containing harmful bacteria (Coliform and E.coli) coming from outside (290304 liter/day).
$x_m(t)$ is the mPHO containing phototropic bacteria from J-Biotech schedule (liter/day).

The mathematical model for three competing species with immigration is the following system:

$$\dot{x}_1 = (k_1 - k_2 x_1(t) - k_3 x_2(t) - k_4 x_3(t))x_1(t) + k_5\left(\frac{x_m(t)G^*}{V}\right) \quad (1)$$

$$\dot{x}_2 = (k_6 - k_7 x_1(t) - k_8 x_2(t) - k_9 x_3(t))x_2(t) + k_{10}\left(\frac{x_p(t)BE^*}{V}\right) \quad (2)$$

$$\dot{x}_3 = (k_{11} - k_{12} x_1(t) - k_{13} x_2(t) - k_{14} x_3(t))x_3(t) + k_{15}\left(\frac{x_p(t)BC^*}{V}\right) \quad (3)$$

$x_1(0), x_2(0),$ and $x_3(0)$ from data at CP1.

4.0 NUMERICAL RESULTS

In order to get all the required parameters k_1 to k_{15}, we substitute the left-hand side of the ODE's by finite difference approximation method. We use the 5-point central difference method as follows [10] to get the parameters in Table 3.

$$f'(t) \approx \frac{1}{12}\left[-f(t+2h) + 8f(t+h) - 8f(t-h) + f(t-2h)\right].$$

Table 3 The parameters determined by 5-point central difference parameters of the proposed model based on the data given

k_1=0.52501600	k_9=1.90226×10^{-6}
k_2 = 0.0000219	k_{10}=0.0149868
k_3=-6.69867×10^{-6}	k_{11}=1.3176200
k_4=5.79328×10^{-6}	k_{12}=0.0002464
k_5=0.04578800	k_{13}=-3.31719×10^{-6}
k_6=1.38664000	k_{14}=3.43005×10^{-6}
k_7=0.00025591	k_{15}=0.0497970
k_8=-1.47168×10^{-6}	

Using the above parameters, we solve the mathematical model and make a comparison between the experimental data at CP2 and our numerical results as shown in Figure 4. It shows that bacteria E.coli and Coliform from our mathematical model practically follow the given experimental data at CP2 with PSB according to the given schedule. This validates our mathematical model.

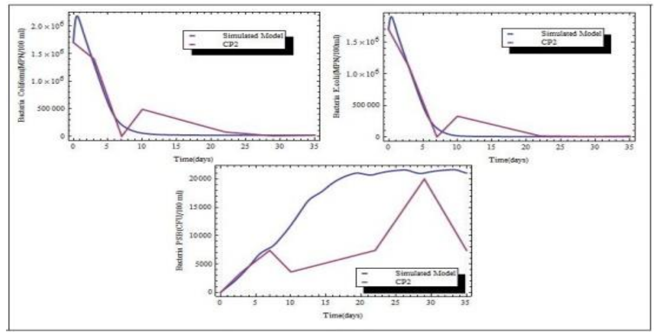

Figure 4 Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic) at CP2. The blue curve shows the numerical results and the red curve shows the the experimental data

We next use our mathematical model to make predictions what happen to the bacteria population with several different treatment schedule of mPHO application. The simulations are shown in Figures 5-8 which are based on applying mPHO at 10 liters/day, 30 liters/day, 50 liters/day, and 0 liter/day.

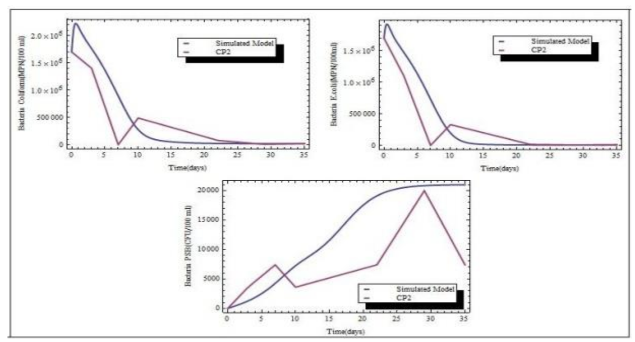

Figure 5 Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic) at CP2. The blue curve is based on the simulation of applying 10 liters per day of mPHO. The red curve is based on the original experimental data

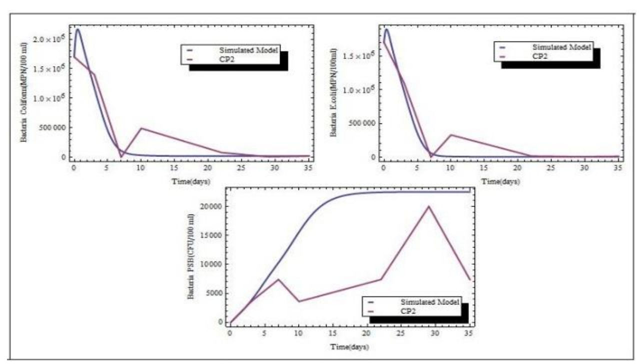

Figure 6 Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic) at CP2. The blue curve is based on the simulation of applying 30 liters per day of mPHO. The red curve is based on the original experimental data.

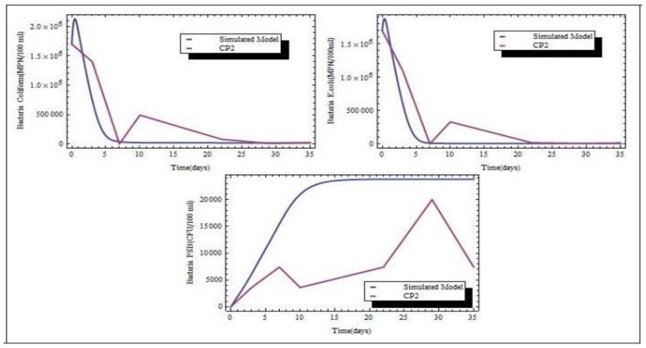

Figure 7 Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic) at CP2. The blue curve is based on the simulation of applying 50 liters per day of mPHO. The red curve is based on the original experimental data

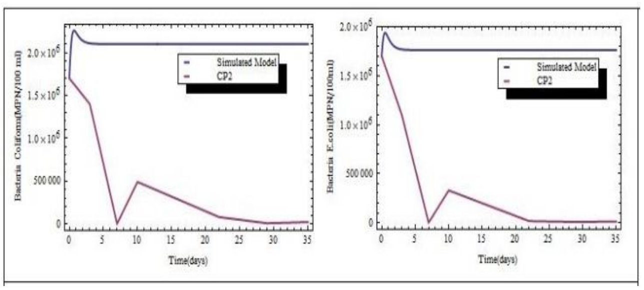

Figure 8 Graphs of comparison between the population of bacteria (E.coli, Coliform and Phototrophic) at CP2. The blue curve is based on the simulation of applying 0 liters per day of mPHO, i.e. no application of mPHO. The red curve is based on the original experimental data

The aforementioned Figures 5-8 show the applications of different amount of mPHO. Table 4 shows the values of mPHO applied and the corresponding cost as well as the amount that could be saved. From the current schedule, the total amount of mPHO is 575 liters. This amount is reduced to 350 liters when we applied 10 liters per day mPHO. If we change the current schedule to 30 liters per day, the total amount of mPHO becomes 1050 liters which is more than the current schedule. If we increased the application of mPHO to 50 liters per day the total amount of mPHO is increased to 1750 liters which is actually a waste of mPHO. However, according to Figure 8 when we do not use mPHO at all, the populations of E.coli and Coliform do not get reduced.

Table 4 The parameters of the proposed model based on the data given

Schedule	Cost	Save
Current	575	0
10 liters per day	350	225
30 liters per day	1050	-475
50 liters per day	1750	-575

According to Table 4, changing the current schedule to 10 liters per day can save 225 liters of mPHO. Therefore, the table suggests that the schedule of 10 liters per day is better than the current schedule. In addition, applying the schedule of 30 liters per day or even 50 liters per day may not change significantly the population of the bacteria.

5.0 CONCLUSION

In this paper a mathematical model based on a system of ordinary differential equations for wastewater treatment process of oxidation pond has been developed. Then, a set of unknown parameters were obtained for this model by considering the real data taken from oxidation pond. These parameters were used in several simulations where the numerical results show that the model can well-predict the behaviour of the micro-organisms involved in the pond. The proposed model also shows the effectiveness of mPHO for degradation of pollutant (E.coli and Coliform). It is also noted that by using a certain amount of mPHO we can reached to the maximum decontamination without having excessive amount of mPHO. This result can be used to construct a new schedule to be applied in the future.

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