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MATHEMATICAL ERRORS IN ADVANCED CALCULUS: A SURVEY AMONG ENGINEERING STUDENTS

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ABSTRACT

Mathematics is widely used in all areas of learning. Students who do not master the basic mathematics will face many problems in learning at the advanced level. This study was conducted to identify the mathematical errors and misconceptions in advanced calculus. The related topics are differentiation of two variables, method of integration, and differential equation. A sample of 80 students' final examination answer sheets for the subject of MAT235 was closely examined to identify the mathematical errors and misconceptions. The technique used to collect the data is quantitative method using descriptive analysis. The findings of this study showed that most of the students were still weak in mastering mathematical basic facts and concepts. The weakness in understanding the basic concepts may lead them to use the wrong strategy when solving mathematics questions. This misconception should be identified by the lecturers and rectified by the students in order to enhance their learning in higher education.

Keywords: Mathematical error; calculus; misconception; education; descriptive analysis.

1. INTRODUCTION

Mathematics is a science that greatly influences daily lives and forms the basis of many disciplines including the engineering world. Therefore, mathematics education is very important for students to understand and master the analytic skills, especially at the tertiary level. Thus, various studies conducted by academics have reported a wide range of learning difficulties in mathematics at the advanced level (Siti Aishah and Noor 'Aina, 2005; Muzangwa and Chifamba, 2012; Suryanto, 2014). Mathematical errors that are often seen include basic mathematical error, method of solving problem, and using the wrong formula. In addition, students often make mistakes in interpreting questions and doing the solution using the wrong method (Siti Aishah and Noor 'Aina, 2005).

In Universiti Teknologi MARA (Penang), the mathematics course that is offered in the second and third semesters is calculus. Calculus is a branch of mathematics that is important and widely used at the undergraduate level. Superficial knowledge of calculus concepts will affect the mathematical knowledge at the tertiary level. This is because calculus can help students to

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build their knowledge in other mathematical areas. Sofronas (2011) had found that the mistakes are often made by the first-year calculus students. She found out that the students were either weak in the mastery of calculus concepts or calculus fundamental skills. Students were not able to establish the links between and among concepts and skills, and they also lacked of the ability to use the fundamental calculus ideas. Therefore, these make students difficult to understand the topics of advanced calculus.

According to Muzangwa and Chifamba (2012), majority of errors in calculus are lack of basic knowledge in algebra. Students' mistakes are said to be the indicator of their basic knowledge gained from the early stage. This situation should not be ignored; if it is not handled properly, it will cause further difficulties in understanding mathematics at the next level. Hence, misconceptions and errors committed by students need to be identified and retified so that the same types of errors do not recur.

The method used in this study is to classify mathematical errors based on the classification of Radatz (1979). According to the study, Radatz suggested four technical aspects of errors in learning that covers all topics in mathematics, which are: (1) difficulties in gathering information from questions, (2) deficiency in requisite basic skills of facts and concepts, (3) incorrect associations or rigidity, and (4) application of rules or strategies that are not relevant. The classification was chosen because it is more clarified and appropriate to the written type questions. The classification of Radatz had also been employed by Siti Aishah and Noor' Aina (2005) in their studies on the students' level of ability in basic mathematics.

2. METHODOLOGY

The sample taken for this study consists of 80 diploma engineering students. They took the examination for the course MAT235 (Calculus II for Engineering) in the third semester. There are five key questions on different topics in the examination paper. Questions drawn from this study are related to the topics of differentiation with two variables, method of integration, and differential equation. All questions were reviewed and revised. Each type of mathematical errors made by the students was determined. Table 1 shows the details of the questions used in this study.

Table 1: Main components of the test

Components	Question No.
Method of Integration	Q2, Q3b, Q5b
Differentiation with Two Variables	Q1, Q3a, Q4a, Q5a
Differential Equation	Q3c, Q4b, Q5c

Each component of the test examines the concepts and skills as follows:

(a) First Component (Method of Integration):

Solve the integration with the correct method: Improper integral, integration of rational function, integration of trigonometric substitution, and integration of trigonometric function.

(b) Second Component (Differentiation of Two Variables):

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Find the limit using L'hopital's rule; find the extremum, related rate and approximation problem; and find the equation of a tangent line using partial fractions to obtain the slope of the graph.

(c) Third Component (Differential Equation):

Solve the differential equation with the proper methods of linear and homogeneous equations; Solve problems using the separable equation method in producing models for answering questions.

Mathematical errors that have been identified are classified (Radatz, 1979) as follows:

1. Difficulties in gathering information from questions

   Students do not interpret the facts given in the problem accurately. Students may not understand the facts given, which cause them make mistakes when solving problems.

2. Deficient mastery of basic facts and concepts

   Students use incorrect concepts and do the wrong basic operation. They make mistakes in the aspects of arrangement and use of improper operation.

3. Incorrect association or rigidity

   Students use the correct method but they do not relate exactly the correct concepts.

4. Application of irrelevant rules or strategies

   Students tend to create their own laws or rules. They interpret the question correctly but the procedure or the strategy to solve the problem is inaccurate or irrelevant.

Descriptive analysis was used to determine the percentage of students involved in each classification.

3. RESULTS AND DISCUSSION

The study found that the classifications of errors obtained in the final examination are as follows:

1) Difficulties in gathering information from questions

Students interpreted the facts in the questions inaccurately. Students did not understand the facts given which caused them to make mistakes when solving problems.

Question 2a: Evaluate

$$\int_1^{\infty} \frac{2}{(x+1)^3}\,dx$$

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Student's Solution:

$$\int_1^{\infty} \frac{2}{(x+1)^3}\,dx = 2\left[\ln (x+1)^3\right]_1^{\infty}$$

$$\text{Let } t = \infty$$

$$= 2\left[\ln (x+1)^3\right]_1^{t}$$
$$= 2\left[\ln (t+1)^3 - \ln(t+1)^3\right]$$

Comment: Students applied incorrect method of integration. Students just considered the answer was natural logarithm when *x* was in the denominator. The correct method should be substitution method for integration. Overall, they are supposed to use the formula for integration of unbounded improper function that include limit and integration.

2) Deficient mastery of basic facts and concepts

When students showed the arrangement or usage of the concept, they made the mistake on the aspects of the arrangement and use improper operation.

Question 2b:

(i) Given

$$\frac{x^2+4x-1}{2x^3+x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

find the values of A, B and C.

(ii) Hence, find

$$\int \frac{x^2+4x-1}{2x^3+x^2}\,dx$$

Student's Solution:

(i)

$$\frac{x^2+4x-1}{2x^3+x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$
$$x^2+4x-1 = Ax(2x+1) + B(2x+1) + Cx^2$$

$$Ax(2x+1) = x^2$$
$$B(2x+1) = 4x$$
$$Cx^2 = -1$$

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Comment: Students applied the correct method of partial fraction. However, they made a mistake when comparing the coefficients. Students have to expand the equation before comparing the coefficients.

Student's Solution:

(ii)

$$\int \frac{x^2+4x-1}{2x^3+x^2} = \int \frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}\,dx$$
$$= A\ln x - Bx^{-3} + C\ln(2x+1)$$

Comment: Students did not integrate the expression properly and they used the incorrect method of integration. The correct method to solve the third term is to use *u* as a substitution, but students just considered the answer was a natural logarithm when *x* was in the denominator.

3) Incorrect association or rigidity

Students used the correct method but they did not exactly associate the correct concept.

Question 3b: Find

$$\int \frac{x}{\sqrt{x^2+9}}\,dx$$

by substituting $x = 3\tan\theta$.

Student's Solution:

$$\sqrt{x^2+9} = \sqrt{(3\tan\theta)^2+9}$$
$$= \sqrt{9\tan^2\theta+9}$$
$$= \sqrt{9(\tan\theta+1)}$$
$$= \sqrt{9\sec^2} = 3\sec\theta$$

$$\int \frac{x}{\sqrt{x^2+9}}\,dx = \int \frac{3\tan\theta}{3\sec\theta}(3\sec^2\theta)\,d\theta$$
$$= \int \frac{3\tan\theta}{\sec\theta}d\theta + \int \sec^2\theta\,d\theta$$
$$= \int 3\tan\theta\cos\theta\,d\theta + \tan\theta$$

Comment: Students used the correct method of integration by trigonometric substitution, but the concept of integration was wrong. Students made a mistake when changing the multiplication to additional operation and they also did not simplify the operation properly. Students should simplify the operation first and use a product of tangent and secant method.

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4) Application of irrelevant rules or strategies

Students tended to create their own laws or rules. They interpreted the question correctly but the procedure or the strategy to solve the problem was inaccurate or irrelevant.

Question 5b: Find

$$\int \cos^2\theta(1+\cos\theta)\,d\theta$$

Student's Solution:

$$\int \cos^2\theta + \cos^3\theta\ d\theta$$
$$= \int (\sin^2\theta - 1) + \cos^3\theta\ d\theta \qquad \text{let } u = \sin\theta$$
$$du = \cos\theta\, d\theta$$
$$= \int (u^2 - 1) + \cos^3\theta\ \frac{d\theta}{\cos\theta}$$
$$= \int (u^2 - 1) + \cos^2\theta\ d\theta$$

Comment: At the beginning, students did the correct step, but they did not use the correct formula to solve the additional function. The proper solution method is to integrate the expressions separately using the different methods to obtain the solution, but the students combined the expression and solved it by using u substitution only.

Explanations and examples above are some of the errors that have been identified. There are many other mistakes that have been made by students when answering the final exam questions. Table 2 shows the percentage of errors made by the students depending on the classification of errors.

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Table 2: Results on the errors made based on the classification

	Classification of error				Total
Question	1	2	3	4	(%)
Q1a(i)	4	4	18	1	3.6
Q1a(ii)	12	15	10	15	6.9
Q2b	4	22	17	8	6.7
Q2a	4	22	17	8	6.7
Q2c	3	57	2	1	8.3
Q3a	6	0	20	0	3.4
Q3b	5	25	1	13	5.8
Q3c	43	15	11	5	9.8
Q4a	5	2	17	37	8.0
Q4b(i)	22	9	8	29	9.0
Q4b(ii)	10	25	4	15	7.1
Q5a	13	6	29	4	6.9
Q5b	10	18	4	29	8.0
Q5c	4	6	23	41	9.8
Total(%)	19	30	24	27	100

There are 758 errors detected from MAT235 final examination. As shown in Table 1, the highest percentage of error made by students as 9.8% falls under the topic of differential equation (Questions 3c and 5c) for This is probably because it is a new topic introduced in the third semester as compared to the other topics. Therefore, students need to master a lot of formulae and solutions that are appropriate to the questions. The results also show that the highest number of errors committed by the students is the 57 errors when answering Question 2c. The question covers the topic of integration by parts. Students were found to be very weak in doing the solution. They did a lot of mathematical errors when trying to differentiate and integrate the given functions especially involving fractions. This shows that the students are still weak on the foundation of integration and differentiation that have been studied in the second semester.

Based on the classification of errors, the most committed error by the students is on the second classification using incorrect basic facts and concepts to answer the questions (30%). Students also applied irrelevant rules or strategies to solve the problem (27%). These two classifications are important when learning mathematics at the advanced level. Students will face difficulties when pursuing their studies if they have poor mathematics background (Siti Aishah and Noor' Aina, 2005). Besides, students also did not understand the facts given (19%) and could not associate the information given with accurate concepts when solving questions (24%).

4. CONCLUSION

This study shows that although the students were able to continue their studies in the third semester of calculus courses, they still made many mistakes in calculus. Based on the study, most of the students were still weak in mastering basic facts and concepts. This situation is likely happened because they did not fully understand the basic lessons in mathematics at secondary schools and they understood the concept of mathematics in their own way without

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following the right channel of calculation. The weakness in understanding the basic concepts may lead them to use the wrong strategies when answering the questions. This error should be corrected in order to facilitate the learning of mathematics at the tertiary level.. Lecturers should also play their role in mastering the basic concepts themselves as students always refer to their lecturers. In addition, lecturers also need to attract students to learn mathematics by using the latest technology. One of the examples is using e-learning with making interactive PowerPoint and using calculating software (Maple, Mathematica and Matlab). This can help students to write a clear step-by-step solution with colour effects, animations, and transitions effectively. When students are interested in mathematics, they will seek to understand mathematics in earnest. Indirectly, they will be able to master the basic concepts of mathematics.

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