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DIVINE PROPORTION IN MODERN ARCHITECTURE: A CASE STUDY

Marina Mohamed[a*], Nor Fadhilah Dzulkifli[a] Nazirah Ramli[a], Kamisah Ariffin[b], Muhamad Sahizan Hashim[a]

[a]Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA Pahang, 26400 Bandar Jengka, Pahang, Malaysia
[b]Academy of Language Studies, Universiti Teknologi MARA Pahang, 26400 Bandar Jengka, Pahang, Malaysia

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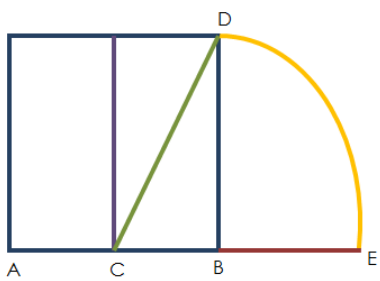
*Corresponding author
arine@pahang.uitm.edu.my

Graphical abstract



Abstract

Mathematics and architecture are interrelated. A building with the appropriate mathematical elements in its design will be impressive in its appearance. One of the most widely used mathematical proportions in architecture is divine proportion, or better known as φ ($\varphi = \lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.618$ with F_n the *n*-th Fibonacci sequence). This type of proportion exists everywhere in nature, including in human body, shapes, artworks, music, paintings and even in the universe. In this study, we investigate the existence of divine proportion in modern architecture. In identifying the divine proportion, geometrical analysis was applied by measuring the dimensions of parts on the floor plan of a building in a higher learning institution in Malaysia. The length, width, height and angles of the floor plan were examined. The results show that the divine proportion, which was obtained from the proportion of length and width, exists in 33% of parts of the first and ground floors of the building examined.

Keywords: Divine proportion, modern architecture

1.0 INTRODUCTION

Proportion can be found everywhere in nature, such as in human body, shapes, artworks, music, paintings and even in the universe. In architecture, proportion is defined as the relationship between one part of a design and another part, or to the whole design, that creates a harmonious relationship [1]. Stakhov [2] states that the proportion and architecture are interrelated and are used as the foundation of standard measurement scale and system of architectural units. The proportion that has uniqueness and beauty is known as divine proportion or golden section, with the value of the proportion approximately at 1.618 and it is known as the most beautiful proportion [3]. This divine proportion has been applied in many remarkable works of sculpture, painting and art. The fundamental principle of all formation is striving for beauty and realities of nature, and, thus, the field of arts comprises divine proportion [4].

Divine proportion has been widely used in architecture since a long time ago. According to Padovan [5], proportion is one of the main principles used in architecture. Obara [6] and Flecther [7] suggest that divine proportion and other mathematical principles exist in architecture. The divine proportion makes the physical appearance of the building aesthetically pleasing. The Renaissance artists used the divine proportion largely in their paintings and sculptures. It can create balance, symmetry and aesthetical pleasantness [8],[9],[10]. Some famous architecture have applied divine proportion in their buildings such as The Great Pyramid of Egypt, Parthenon [11], Great Mosque of Kairouan [12], The Cathedral of Milan, and the Round Temple of Baalbek. This proportion also exists in certain traditional architecture such Iranian house [1], and Palladio's Villa Emo in Italy [13].

Divine proportion is also found in modern architectures, such as in the Pastoor Van Ars church in The Hague [14], The Farnsworth House designed by Ludwig Mies van der Rohe, CN Tower in Toronto, Canada, the United Nations building and the Mexico City Metropolitan Cathedral. These buildings show that the existence and importance of divine proportion in designing modern architecture.

In Malaysia, some of the traditional and modern architecture have also been influenced by the concept of divine proportion. Divine proportion has been used in architecture to introduce aesthetic elements to a mathematical concept. Mohamed *et al*. [15] suggests the existence of divine proportion in traditional Malay architecture while Pathmanathan [16] found the existence of golden section in the geometrical plan and minaret of mosques. These findings show that divine proportion concept has also been applied to construct buildings in the Malay architecture. In Malaysia, research on the usage of divine proportion in designing buildings or houses have been scanty. There is a lot of space to be explored and investigated in determining the usage of this proportion in the Malay architecture. In this paper, the investigation of divine proportion is carried out in one modern building in a higher learning institution in the East Coast of Malaysia. The investigation focused on the floor plan and some other elements of the building. The findings of this study will contribute to the knowledge of divine proportion in the Malay architecture.

1.1 Preliminary Studies

1.1.1 Fibonacci Sequence

The Fibonacci sequence is a series of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, …Each term of the sequence can be obtained by adding two consecutive terms. Mathematically, the sequence can be represented as.

$$F_n = \begin{cases} 1 & n = 1 \\ 1 & n = 2 \\ F_{n-1} + F_{n-2} & n \geq 3 \end{cases}$$

The ratio of the two terms converges to an irrational number which is named as divine proportion and denoted as $\varphi \approx 1.618$.

1.1.2 The Geometrical Construction of Divine Proportion

The construction of divine proportion φ can be obtained from a square rectangle. Boussora and Mazouz [12] proposed a geometrical procedure (Figures 1 and 2) in obtaining the divine proportion. The procedures are given as follows:

Step 1 : Draw a square with AB as its width and BD as its length.
Step 2 : Divide the square into two equal regions with C as the midpoint of AB.
Step 3 : Draw a straight diagonal line of C to D.
Step 4 : Extend the line of CB straight until E, where the length of CE must be the same with the length of CD
Step 5 : Draw a curve, starting from point D to E.

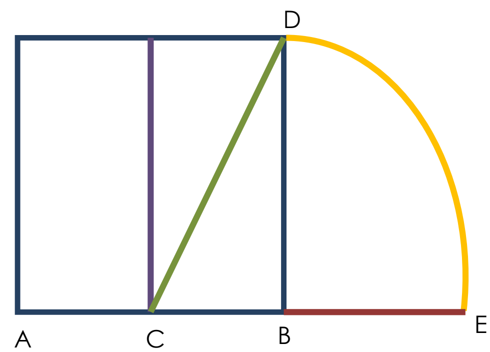


Figure 1 Divine proportion construction

The proportion can be clearly illustrated by line ABE in Figure 2 below.

Figure 2 Divine proportion

Based on Figure 2, if the proportion between r and s approximates to 1.618, then, the ratio is known as divine proportion. The proportion can be calculated as below.

$$\varphi = \frac{r}{s} = \frac{r+s}{r} \approx 1.618.$$

2.0 METHODOLOGY

This study was conducted at a modern building located in a higher learning institution in the East Coast of Malaysia. The interior parts of the building were the main focus of the research. In order to identify the divine proportion, geometrical analysis was applied by measuring the dimensions of parts on the floor plan which were length, width, height and angles. Besides referring to the floor plan, the researchers also took the initiative to conduct self-measurement in ensuring the validity of the dimensions. Figures 3 and 4 show the floor plan of the building investigated.

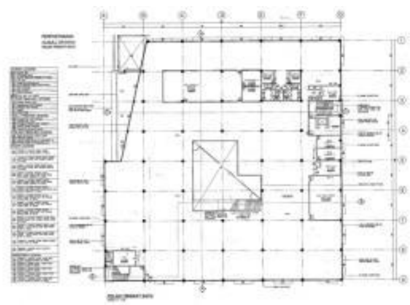


Figure 3 First floor plan of the building investigated

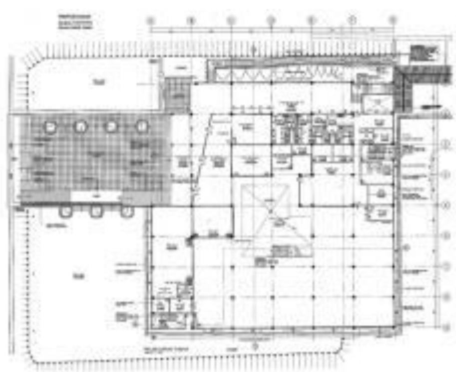


Figure 4 Ground floor plan of the building investigated

3.0 RESULTS AND DISCUSSION

The findings are presented based on the types of analysis carried out. The first section will be on the results of geometrical analysis, followed by the measurement analysis.

3.1 Geometrical Analysis

Based on the first floor plan, the main space is exactly located within the grid of eight rows and five columns as shown in Figure 5. This grid is called Fibonacci rectangle grid, where 5 and 8 are the elements of Fibonacci sequence as discussed by Llebrez and Fran [14]. The grid point represents the pillars of the building investigated. This grid of point is also feasible to any plan of the building (ground floor and first floor).

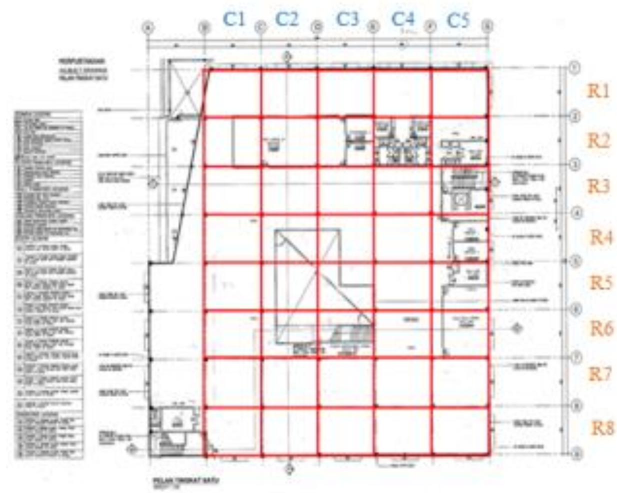


Figure 5 Fibonacci rectangle grid (R8xC5)

In order to verify the presence of the divine proportion, first, the golden rectangular of the ground floor plan was obtained. The geometrical analysis as proposed by [12] was used in obtaining the golden rectangular. The geometrical analysis started by drawing a square ABDF on the floor plan with AB as the width and AF as the length with AB 30 meter (as shown in Figures 6a and 6b). Then, the square was divided into two equal regions with C as the midpoint of AB. By drawing a straight diagonal line from C to D with length CD obtained as 33.54 meters, we extended the line CB to E with equal length of CE and CD. Then, a curve was drawn from point D to E. Therefore, the length of BE is 18.54 meters (since CB is 15 meters). The proportion can be calculated as in Figure 2 whereby AB is 30 meters and BE is 18.54 meters. Thus, we have

$$\varphi = \frac{AB}{BE} = \frac{30}{18.54} \approx 1.618.$$

The ratio of length to the width of the rectangle is approximately 1.618. From this analysis, the golden rectangle is found and it fits into the dimensions of the ground plan of the building investigated.

Figure 6a Golden rectangle (horizontal side) on the ground floor plan

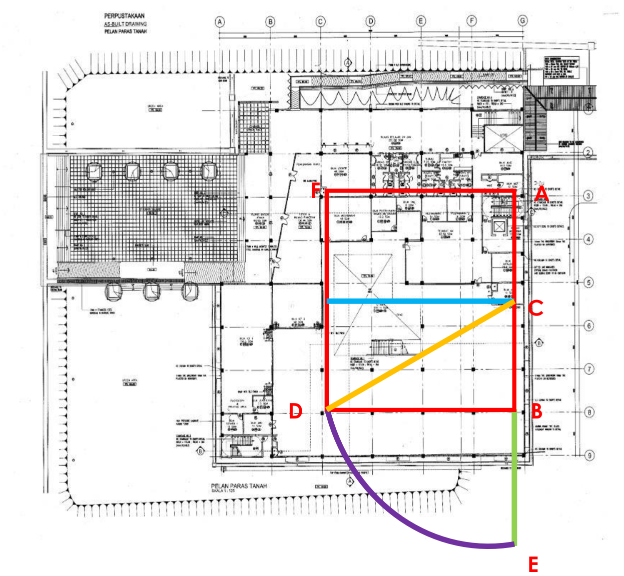


Figure 6b Golden rectangle (vertical side) on the ground floor plan

3.2 Measurement Analysis

In this section, the findings were obtained by using the measurement of the elements and parts of the building investigated. The parts and elements of the buildings can be denoted as:

A_n where $1 \leq n \leq 11$
B_n where $1 \leq n \leq 13$
C_n where $1 \leq n \leq 17$

A_n, B_n and C_n represent the parts in the first floor, the parts in the ground floor and the elements in the building respectively.

The elements of the building measured were the dimensions of the doors, windows and staircase. The divine proportion can be identified by the ratio between the length, width and the height. For any proportions that fall in the interval [1.6, 1.667], the proportion is considered as having divine proportion property.

The findings for parts of the first floor of the building are shown in Table 1. Table 1 shows the proportion of length and width, length and height and, width and height of the first floor. The proportion of length and width of the four parts falls in the interval [1.6, 1.667]. The four parts are Units A_8, A_9, A_{10} and A_{11}. This means that the divine proportion exists in these four parts. For other parts, they do not fall in that interval. Similarly, for the proportion of length and width and, width and height, none of the parts falls into that interval. This shows that the divine proportion exists only in small parts of the first floor of the building around 36% only.

Table 1 Measurement on some parts of the first floor

Part of the building	Length(a)/ Width(b)	Length(a)/ Height(h)	Width(b)/ Height(h)	Existence of golden section
Unit A_1	2.833333	5.862068	2.068966	X
Unit A_2	1.333333	1.379310	1.034483	X
Unit A_3	1.333333	1.379310	1.034483	X
Unit A_4	1.333333	1.379310	1.034483	X
Unit A_5	1.335356	3.034483	2.272414	X
Unit A_6	1.086923	2.436207	2.241379	X
Unit A_7	1.875	1.034483	0.551724	X
Unit A_8	1.566667	1.62068*	1.034483	✓
Unit A_9	1.566667	1.62068*	1.034483	✓
Unit A_{10}	1.946169	1.62068*	0.832759	✓
Unit A_{11}	1.946169	1.62068*	0.832759	✓
% of divine proportion	0%	36.3%	0%	

*: falls in interval [1.6, 1.6667],
✓: the divine proportion exists in parts of the building,
X: the divine proportion does not exist in parts of the building.

Table 2 shows the proportion of length and width, length and height and, width and height of the ground floor. Similar to the first floor findings, only four parts have the values of proportion of length and width between 1.6 and 1.667. The four units are Unit B_6, Unit B_9, Unit B_{10} and Unit B_{12}. This means that the divine proportion exists in these four units. Similar to the first floor findings, for the proportion of length and width and, width and height, none of the parts falls into that interval. The divine proportion exists only in 31% of the parts of the ground floor.

Table 2 Measurement on some parts of the ground floor

Part of the building	Length(a)/ Width(b)	Length(a)/ Height(h)	Width(b)/ Height(h)	Existence of golden section
Unit B_1	1.3333333	2.7586207	2.0689655	X
Unit B_2	1.1428571	1.3793103	1.2068966	X
Unit B_3	1.1428571	1.3793103	1.2068966	X
Unit B_4	1.3333333	1.3793103	1.0344828	X
Unit B_5	1.3333333	2.7586207	2.0689655	X
Unit B_6	1.0609482	1.62068 *	1.5275862	✓
Unit B_7	1.0869231	2.4362069	2.2413793	X
Unit B_8	1.1113333	1.1496552	1.0344828	X
Unit B_9	1.5666667	1.62068 *	1.0344828	✓
Unit B_{10}	1.5666667	1.62068 *	1.0344828	✓
Unit B_{11}	1.9015115	1.3448276	0.7072414	X
Unit B_{12}	1.1414634	1.61379 *	1.4137931	✓
Unit B_{13}	1.3333333	2.7586207	2.0689655	X
% of divine proportion	0%	30.8%	0%	

*: falls in interval [1.6, 1.6667],
✓: the divine proportion exists in parts of the building,
X: the divine proportion does not exist in parts of the building.

The existence of divine proportion was also investigated based on the measurement of some elements in the building. The measurement of width, length and ratio of length and width are presented in Table 3. Only one element (5.9%), which is Unit C_{13}, satisfies the divine proportion properties that falls in the interval between 1.6 and 1.667. Other elements do not satisfy the divine proportion properties.

Table 3 Measurement on some elements of the building

Element	Width (a)	Length (b)	b/a	Golden Section
Unit C_1	1500	2100	1.4	X
Unit C_2	1200	2100	1.75	X
Unit C_3	900	2100	2.333333	X
Unit C_4	900	2100	2.333333	X
Unit C_5	1000	2100	2.1	X
Unit C_6	750	2100	2.8	X
Unit C_7	900	2100	2.333333	X
Unit C_8	600	2100	3.5	X
Unit C_9	1500	2100	1.4	X
Unit C_{10}	900	2100	2.333333	X
Unit C_{11}	545	1130	2.073394	X
Unit C_{12}	600	1130	1.883333	X
Unit C_{13}	1800	3000	1.666667*	✓

Element	Width (a)	Length (b)	b/a	Golden Section
Unit C_{14}	600	1800	3	X
Unit C_{15}	600	1200	2	X
Unit C_{16}	1500	1800	1.2	X
Unit C_{17}	4520	7850	1.7367	X
% of divine proportion			5.9%	

*: falls in interval [1.6, 1.6667],
✓: the divine proportion exists in parts of the library,
X: the divine proportion does not exist in parts of the library.

4.0 CONCLUSION

This paper has investigated the existence of divine proportion in one modern building in a higher learning institution in the East Coast of Malaysia. The geometrical analysis shows the presence of golden rectangle in the floor plan, while the results from the measurement analysis show that the divine proportion exists in 33% of the parts of the first and ground floor of the building. This percentage is about one third of the ratio which indicates that the existence of divine proportion is quite high. It can be considered as having aesthetical pleasing to the eye. At the same time, only 5.9% of the elements possess the character of divine proportion. This shows that the divine proportion exists only in a small element of the building and the existence of divine proportion is very small. It indicates that the elements of the building can hardly be seen as aesthetical pleasing.

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