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Corner Detection Of Outline Images As A Modified Measure Of Eigenvalues Of Covariance Matrix

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ABSTRACT

Many researchers have studied corner detection since few decades ago. Corner points at sharp corners are easy to be detected, but corner points at smooth curves are hard to be detected. Rather than used smaller eigenvalue, ratio of eigenvalue of covariance matrix is used as curvature measure. An automatic determination of region of support is used to find the suitable length of region of support. We implemented the corner detection on Lambda font, a computed tomography image and letter 'Qaf'. A three point moving average was used to eliminate the noise in image. To make sure the corner detection is accurate; accuracy is analyzed based on the ground truth corner points, number of corner points detected and the number of corner points matched to the ground truth corner points. We found that the accuracy of the corner detection is high and the suitable length of region of support and threshold values are determined. We also found that by using λ_R as a curvature measure, the results are better compared to λ_S use as curvature measure.

Keywords: *corner detection; corner points; region of support; ratio of eigenvalue of covariance matrix; curvature measure.*

1. INTRODUCTION

Corner is the sharp end of a curvature in an image. Corner can also be defined as a point where two lines from different angle and direction meet. Many researches on corners have been done since last few decades and the researchers have given the definition of corners based on their approaches. Rangarajan *et al.* [1] stated that a corner is the joint point of two or more straight line edges especially in the features of images. Tsai *et al.* [2] described corners on a curve where two relatively straight line segments intersect and the points on the extracted boundaries with high curvatures. Chiou and Liang [3] described high-curvature points as corners in general. Dey *et al.* [4] defined a corner as a meeting point of two edges, where an edge is a sharp variation in image brightness.

Corners play important roles in computer vision because they are invariant to geometric transformations, such as rotation, translation and scaling as mentioned by Horng *et al.* [5]. The corners will store the characteristics or information of the image. Corner points will show the features of the image when they are joined with lines or curves. By storing just the corners of an image, the storage space and the processing time can be reduced thus more efficient as it

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takes less memory. According to Yeh [6], corners provide much information for shape representation and object recognition. Safraz and Razzak [7] in their research used corner points in outline capturing system. They developed an algorithm to capture the outline of a font. From there, the usage of corner points can lead to various image processing applications such as processing videos.

According to Zhu *et al.* [8], corner detection plays an important role in image processing research and computer vision. Usually, corner detection is the basic process before starting image processing. Therefore, corner detection has a great impact in image processing. Topal *et al.* [9] discussed that corner detection is very important especially in computer vision research because of the strong need for robust key point detection algorithms. Frantz *et al.* [10] and Mohammadi & Fatemizadeh [11] are some of the researchers who studied corner detection to be used in medical purposes. In font generation, Yahya *et al.* [12] stated that there are two methods to create outline fonts which are either by digitizing the ready-designed image of font and then fitting curves to the corners or by designing the outline in the computer.

Generally, corner detection can be classified into two major categories according to detection strategies which are gray-level approaches and boundary based approaches. Gray-level approaches act directly on the image without the need for image segmentation while boundary based approaches act directly on the boundary points that have been extracted from images and then find the changes of the curvature. The high curvature points will be known as corners. Boundary based approaches are stable in detecting polygon corners because the polygon curves are sharp [6]. Gray-level approaches is not good at localizing corners while boundary based approaches is simpler and faster [12]. In our study, we will be focusing on the boundary based approach.

The shape of an image depends on the outline where it may contain straight lines or smooth curves. The images that we are dealing with are 2D images. In this study, the images that will be used are Lambda font, a computed tomography image and the letter 'Qaf'. We chose these three images because they are in different types and have their own characteristics. A Lambda font image consists of well-defined smooth and sharp corners. A computed tomography image has smooth corners that are not so well defined that has undulating curves and tend to be rounded edges and the letter 'Qaf' has large smooth curves especially on the body's part. For letter 'Qaf", we are only interested of its body only without the dots. In our study, we find the corners from the digitized data points of the three images. The outlines of the images are extracted and then, digitized data points are obtained from the outlines. The Lambda font, a computed tomography image and letter 'Qaf' are as shown in Figure 1, 2 and 3 with their digitized data points respectively.

Figure 1: Image of Lambda Font and its Digitized Data Points

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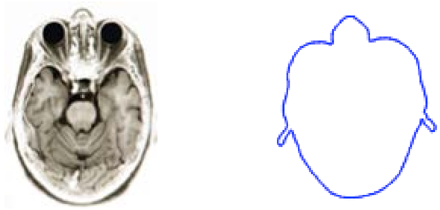

Figure 2: Image of Computed Tomography Image and its Digitized Data Points

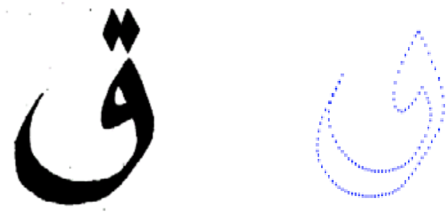

Figure 3: Image of Letter 'Qaf" and Digitized Data Points of the Body of Letter 'Qaf'

Region of support (ROS) is the dominant points on the curve segments bounded by its left and right arms. The purpose of ROS is to focus on small boundary segments in an image. Each point in the image has its own ROS. Parameter k is usually referred to ROS that is the number of pixels on the arms of the ROS. Appropriate length of ROS can eliminate image noise and quantization effect. ROS is not necessarily symmetric. There are researchers using asymmetric ROS. For symmetric ROS, the length can be measured as $2k + 1$. In Figure 4 illustrates example of the points in the ROS. The length of the ROS is 7 because k is 3.

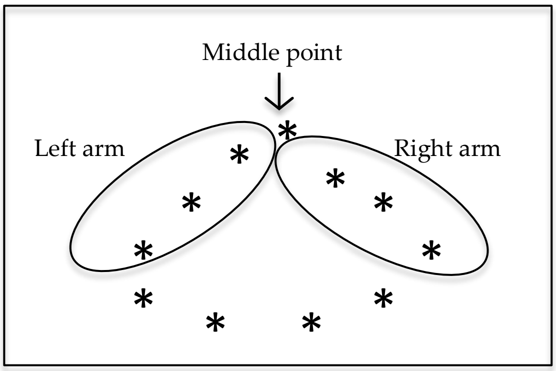

Figure 4: Point in Region Of Support where *k* is 3

The example shown above is a type of single region of support. Since the suitable length for the arm of the ROS is difficult to be obtained, a few researchers have carried out researches about this. Sun [13] proposed a suitable value for k where in his study, called k_p. Zhu [8] have introduced a multi ROS.

From the digitized data points, the corners of the image are obtained by using corner detector. Tsai *et al.* [2] introduced a corner detector by using eigenvalues of covariance matrix of data points over a small ROS to test on object shapes containing various curves and circular arcs. This technique used the smaller eigenvalues to determine the sharp corners and the weak

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corners. In this research, we use a curvature measure using eigenvalues of the covariance matrix of the ROS. Instead of using the small eigenvalue, we utilize the small and large eigenvalues. Our curvature measure is called ratio of eigenvalues of covariance matrix, λ_R where $\lambda_R = \frac{\lambda_S}{\lambda_S + \lambda_L}$.

Covariance matrix in corner detection is a 2×2 matrix consists of elements of variance of the x and y coordinates of the boundary points. The matrix is symmetric and positive semidefinite. From the covariance matrix, the smaller and larger eigenvalues are obtained.

Each point contains its own curvature. For the strong corner, its curvature is higher than the weak corner. In corner detection, we are only interested in determining the corner points that have high curvature which are very sharp in nature and easy to be determined visually. Points that have lower curvature are flat in nature and they are hard to be determined visually on curves. After tresholding, the weak curvature will be removed.

After all the low curvature points are removed, non-maxima suppression is performed on each remaining point. If the point has the highest curvature among the neighbours, it will be chosen as a corner point. This process will continue until all the remaining points are scanned. Hence, the remaining points after non-maxima suppression are the corner points. Since the points are compared to their own neighbours in the same region, thus there is only one corner point that can be detected in the same region. Therefore, choosing the right size of ROS is important.

The objectives of this research are: (a) to eliminate noise on the outline of images, (b) to determine the effect of region of support and threshold in corner detection, (c) to determine corners of outline of images using curvature measure that is ratio of eigenvalues of covariance matrix, λ_R and (d) to analyze the accuracy of the corner detection method and compare it to the usual measure that is the smaller eigenvalue, λ_S.

2. METHODOLOGY

For the corner detection procedure, instead of using the smaller eigenvalue as the curvature measure as suggested by Tsai *et al.* [2], a ratio of eigenvalues is used while the automatic determination of the ROS for corner detection used is a technique as presented by Horng *et al.* [5].

2.1 Smoothen the Sample Points

The procedure is started by smoothing the sample points to eliminate the noises of the digitized data points. Since we are dealing with point coordinates, we have to consider for both x-axis and y-axis by calculating them simultaneously by using

$$x_i = \frac{x_{i-1} + x_i + x_{i+1}}{3}, y_i = \frac{y_{i-1} + y_i + y_{i+1}}{3} \tag{1}$$

where x_i is the current x-coordinate, x_{i-1} and x_{i+1} are the previous and next x-coordinates respectively. Similar to the y-coordinates as explained for x-coordinates.

2.2 Corner Detection Procedure

The eigenvalues of a covariance matrix were used as a measure of significance. This is the main step in this procedure to get the corners of the images. Let P be a closed boundary that is represented by a sequence of n points, $p_i(x_i, y_i)$ for $i = 1, 2, 3, \dots, n$ as explained by Tsai *et al.* [2]. Instead of using the smaller eigenvalue as the curvature measure as suggested by Tsai *et al.* [2], a ratio of eigenvalues is used. The ratio λ_R can be indicated as

$$\lambda_R = \frac{\lambda_S}{\lambda_S + \lambda_L} \tag{2}$$

Since images have variable size of curve region, their regions of support also have different sizes. It is expected that eigenvalues affect the cornerness of the curve segment. We use the ratio because it gives a normalized measure where we assume large values of the data points or large ROS will have less effect on the ratio.

After obtained the λ_R, or we call it as significance measure, the points that have significance measure less than threshold, t will be eliminated. In this study, t is defined between 0.0001 and 0.005.

Next, non-maxima suppression on P is performed to eliminate remaining points after thresholds, which are not local maxima in the ROS for each point.

2.3 Region of Support (ROS)

The automatic determination of the ROS for corner detection used is a technique as presented by Horng *et al.* [14]. This step is an optimizing procedure in determining the ROS automatically. In this procedure, we want to find the optimal value for ROS, k. The value of k will be set in a range until the optimal k^* is found which is when the discriminant criterion f value is at maximum.

At the beginning, we set the k in a search range $[k_1, k_2]$ where in this experiment we used $k_1 = 2$ and $k_2 = 12$. Then, general corner detection procedure is performed starting with $k = k_1$ based on a given t where t are 0.0001, 0.0005, 0.001 and 0.005. After corner points are detected, discriminant criterion f is calculated and will be stored for each k. The optimal k^* is chosen from the corresponding corner detection that has maximum f.

2.4 Performance of the Corner Detection

To ensure the corner detector can work on variety of shape, the corner points were analyzed. In our study, to evaluate the performance of our corner detector, it must satisfy a criteria performance that was introduced by Mohanna and Mokhtarian [15] to measure accuracy. This criterion is based on human judgement. To mark the ground truth corners, we have marked the corners on the three images by visual observation. The corners marked were chosen as ground truth for the images. Then, by comparing the corners detected with the ground truth, we computed the accuracy as follows

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$$ACU = 100 \times [(N_a/N_0) + (N_a/N_g)] \tag{3}$$

where N_0 is the number of corners in original image, N_a is the number of matched corners in original image when compared to ground truth corners and N_g is the number of corners in the ground truth. In other words, N_0 is the number of corner points detected and N_a is number of true corner points detected. Figure 5 show the ground truth corners in Lambda font, computed tomography image and letter ‘Qaf’ respectively. There are 10 ground truth corners in Lambda font, 28 ground truth corners in computed tomography image and 9 ground truth corners in letter ‘Qaf’.

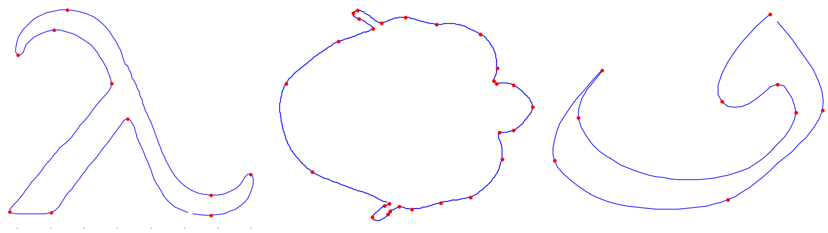

Figure 5: Ground Truth Corners Marked

3. RESULTS AND DISCUSSION

The performance of our corner detection is analyzed based on the performance index [15].

3.1 Data Analysis

Each point has its own value of ratio of eigenvalues of covariance matrix. The value of ratio of eigenvalues of covariance matrix depends on value of ROS k and threshold t. Consequently, when k changes, curvature measure for each point also changes. The smaller the k, the more corner points are detected. However, more corner points detected do not show that the corner detector is good. The more corner detected can lead to more false corners detected. Even on a straight line sometime the detector will detect the noise as corner points because of the staircase effect problem.

The corners detected in the three images are evaluated with different values of parameter k and t. The value for threshold set is at 0.0001, 0.0005, 0.001 and 0.005 and the search range for region of support is $k \in [k_1, k_2]$. Table 1, 2 and 3 below show the criterion of accuracy for the three images.

Table 1: Accuracy of Corner Detector for Lambda Font Image When t is at 0.0001, 0.0005, 0.001 and 0.005

t	**0.0001**	**0.0005**	**0.001**	**0.005**
k	5	3	2	2
ACU	90%	85.71%	76.32%	88.46%

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Table 2: Accuracy of Corner Detector for Computed Tomography Image When t is at 0.0001, 0.0005, 0.001 and 0.005

t	**0.0001**	**0.0005**	**0.001**	**0.005**
k	7	7	7	2
ACU	64.36%	64.36%	67.31%	73.51%

Table 3: Accuracy of Corner Detector for Letter 'Qaf' Image When t is at 0.0001, 0.0005, 0.001 and 0.005

t	**0.0001**	**0.0005**	**0.001**	**0.005**
k	4	4	4	2
ACU	76.19%	76.19%	76.19%	88.89%

From the tables above, we can see that the accuracy of the algorithm for Lambda font image is higher than for computed tomography image. Lambda font image contains many sharp corners while computed tomography image contains many smooth curves. The algorithm detects sharp corners more accurate compare to smooth curves. The highest accuracy for Lambda font is when $k = 5$ and $t = 0.0001$.

As in Table 2, the highest accuracy for computed tomography image is when $t = 0.005$ and $k = 2$. The locations of the corner points detected in the image are at the sharp corners. This proves that the corner detector works best in detecting sharp corners. For letter 'Qaf' also has the highest accuracy when $t = 0.005$ and $k = 2$. The characteristics of computed tomography image and letter 'Qaf' are almost similar because they have large curves in their image. Similar to letter 'Qaf', when k is 2, the sharp corner is detected. Figure 6 shows corners detected of the three images with highest accuracy.

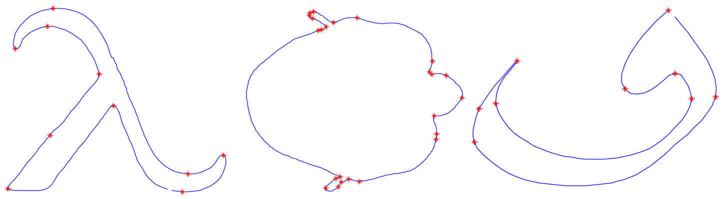

Figure 6: Highest Accuracy of Corner Detected for Lambda Font, Computed Tomography and Letter 'Qaf' Image

3.2 Comparison with Smaller Eigenvalue, λ_S as Curvature Measure

Our research is a modification of the curvature measure that has been introduced by Tsai *et al.* [2] where he used the smaller eigenvalues and we are using the ratio of the eigenvalues. In addition, most technique using eigenvalues of covariance matrix also uses the smaller eigenvalue as their curvature measure. In Table 4 below, we compare the performance of our

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method and Tsai's method. We set same values for thresholds and ROS and then compared only the highest accuracy obtained among the four threshold values for each image. From Table 4, we can see that our approach is more accurate than the smaller eigenvalue. For all the three images, our approach outperforms the Tsai's method. For each image, we used $t =$ 0.0001, 0.0005, 0.001 and 0.005 and range k is between 2 and 12.

Table 4: Results of the Accuracy (%) by Different Approach of Curvature Measure

Type of Curvature Measure	Lambda Font	Computed Tomography Image	Letter 'Qaf'
Smaller Eigenvalue	66.67%	62.50%	76.19%
Ratio of Eigenvalue	90.00%	73.51%	88.89%

3.3 Discussion

From our results, we found that sharp corner points have high curvature measure which in our study we used ratio of eigenvalues as our curvature measure. Our corner detector is able to detect true corner points if we set proper values for threshold and the size of ROS. We have found the most proper values of threshold and the size of the regions of supports for Lambda font image with accuracy of 100%. On the other hand, for computed tomography image, the corner detector detects only either sharp corner points or points on smooth curves. When the threshold is small, the corner detector detects most of the corner points on smooth curves while when the threshold is big; the corner detector detects the sharp corners.

Comparing our curvature measure that is ratio of the eigenvalues of the covariance matrix, λ_R with smaller eigenvalue of the covariance matrix, λ_S that has been introduced by Tsai *et al.* [2], it is found that our curvature measure performs better than Tsai's. For all the three images, the accuracy of corner detector by using λ_R are higher than using the λ_S. Besides that, rather than just using single value of k to find λ_R, this study is automatically calculate λ_R by setting the range of k which are from k_1 to k_2. So, this method can cater for wide range of images that need multi size of k.

4. CONCLUSION

Our study shows that the corner detector managed to detect corners on smooth and sharp curves. However, on images of the category that have smooth curves that are not so well-defined, the detector will detect corners only on one type of segment either on smooth curves or sharp curves segment.

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